



European Research Council

Established by the European Commission

## Slide of the Seminar

# **Restructuring of colloidal aggregates in turbulent flows**

***Dr. Matthäus U. Bäbler***

***ERC Advanced Grant (N. 339032) “NewTURB”  
(P.I. Prof. Luca Biferale)***

Università degli Studi di Roma Tor Vergata  
C.F.n. 80213750583 – Partita IVA n. 02133971008 - Via della Ricerca Scientifica, 1 – 00133 ROMA



ROYAL INSTITUTE  
OF TECHNOLOGY

# Restructuring of colloidal aggregates in turbulent flows

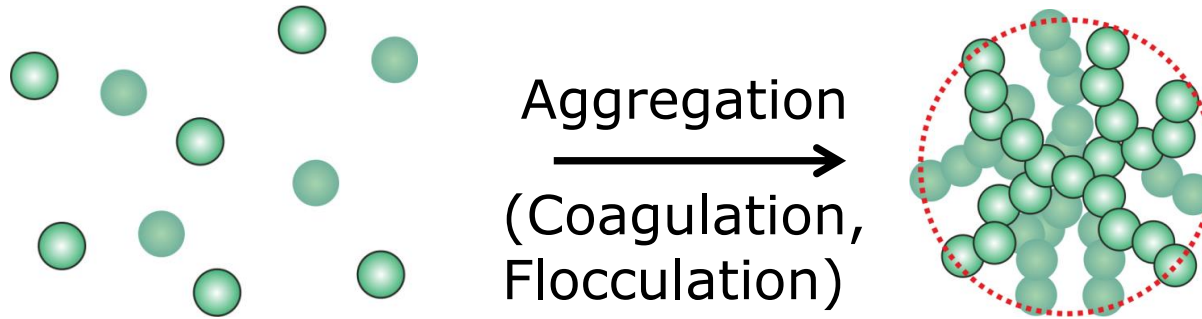
**Matthäus U. Bäbler**

**KTH Royal Institute of Technology, Stockholm, Sweden**

**University of Rome Tor Vergata, Rome, Italy**

**2014-04-15**

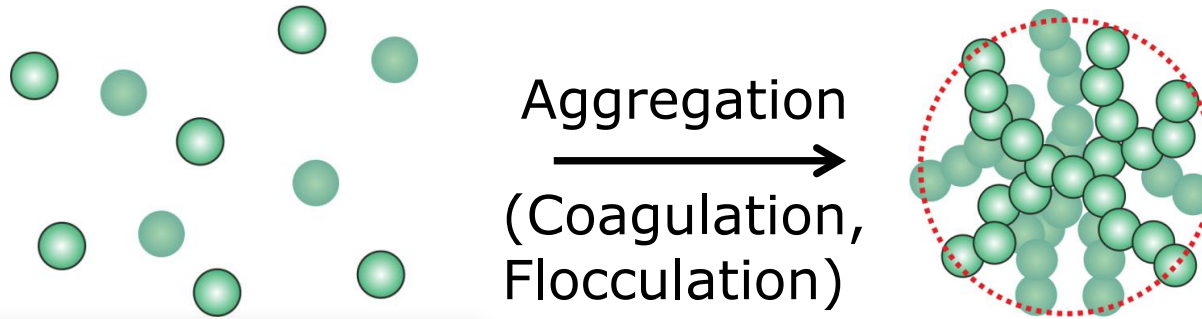
# Aggregation of small particles



Primary particles  
~ (10 nm – 10  $\mu$ m)

Aggregates  
~ up to millimeters

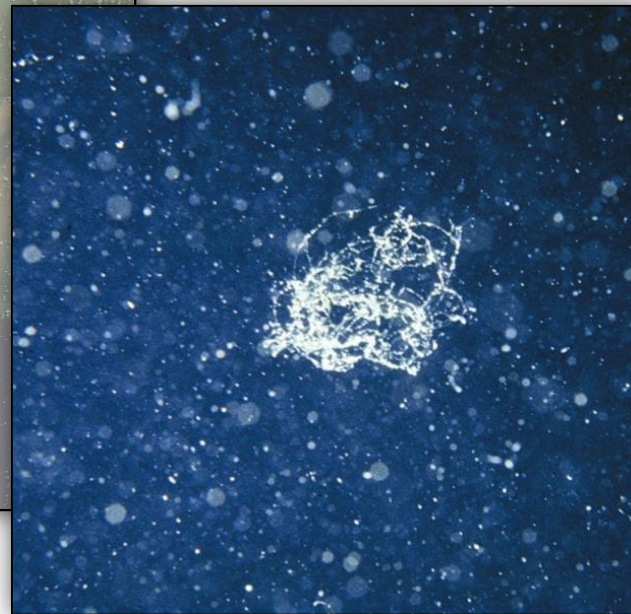
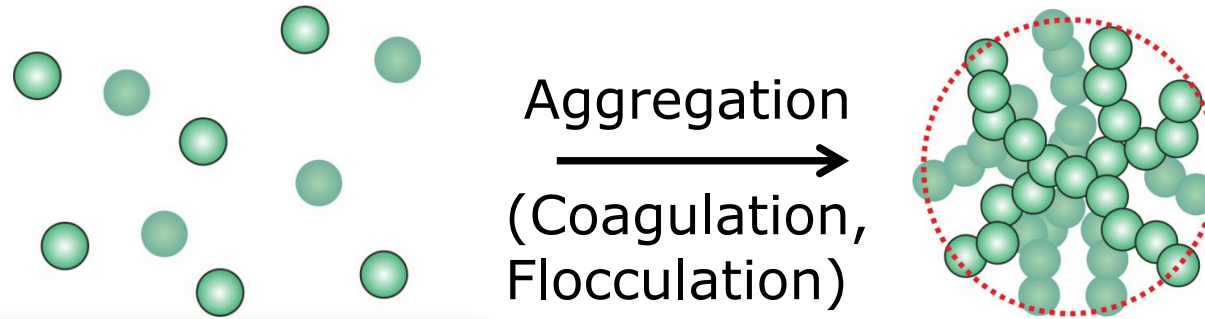
# Aggregation of small particles



Aggregates  
~ up to millimeters

Picture: Satellite image Rio de la Plata Estuary,  
March 10, 2010 ([www.eosnap.com](http://www.eosnap.com), retrieved  
2014-03-12),

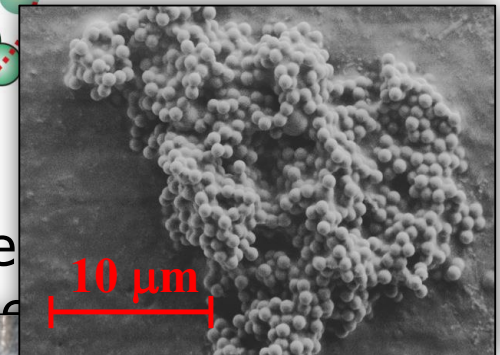
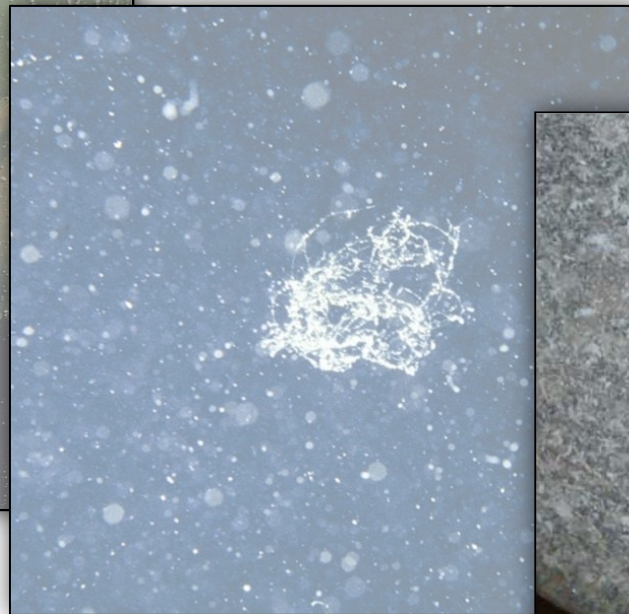
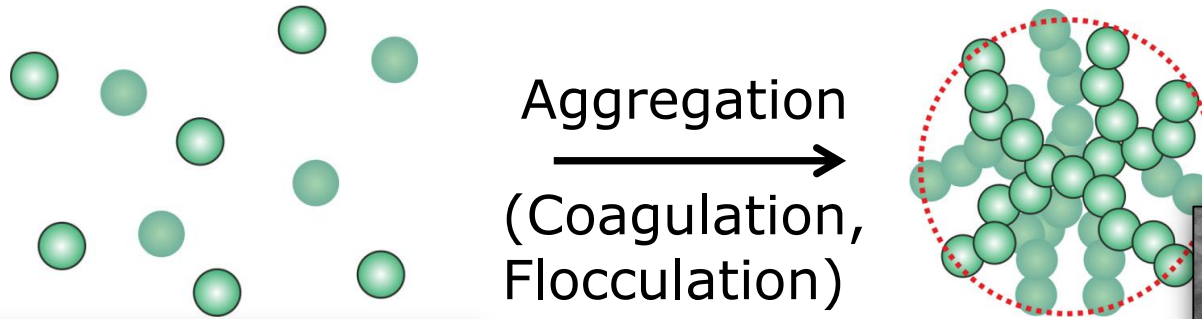
# Aggregation of small particles



aggregates  
up to millimeters

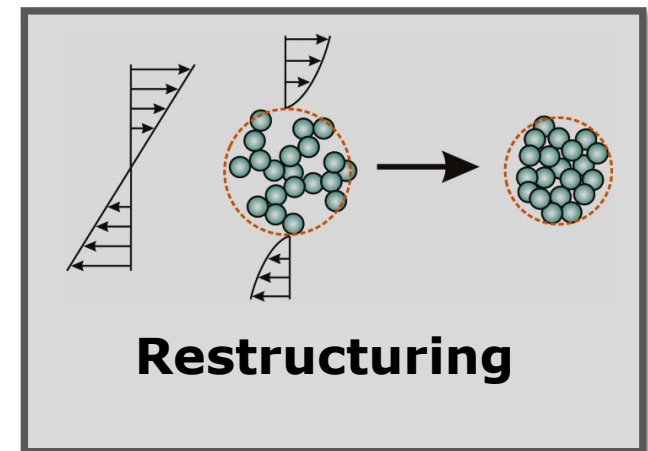
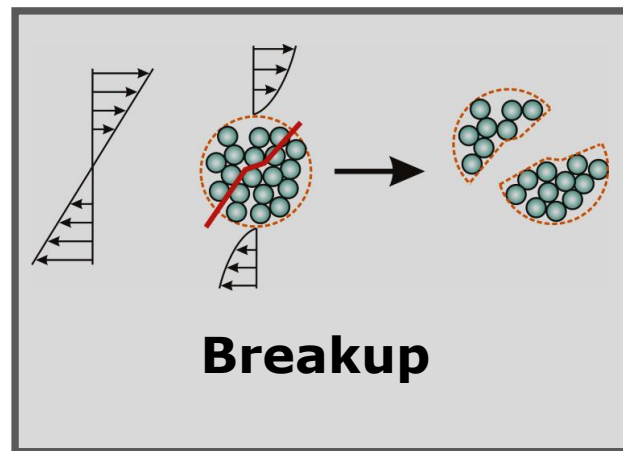
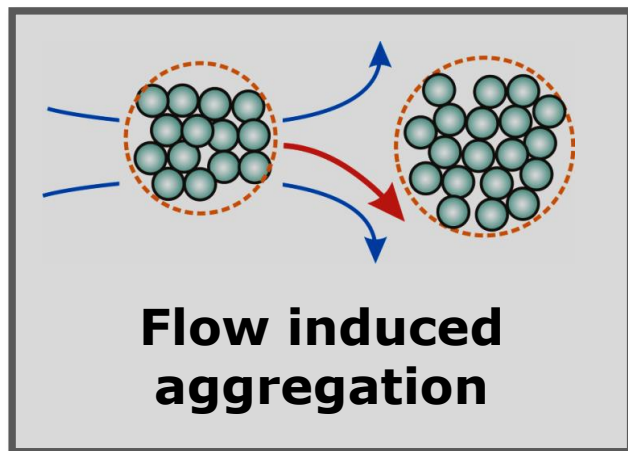
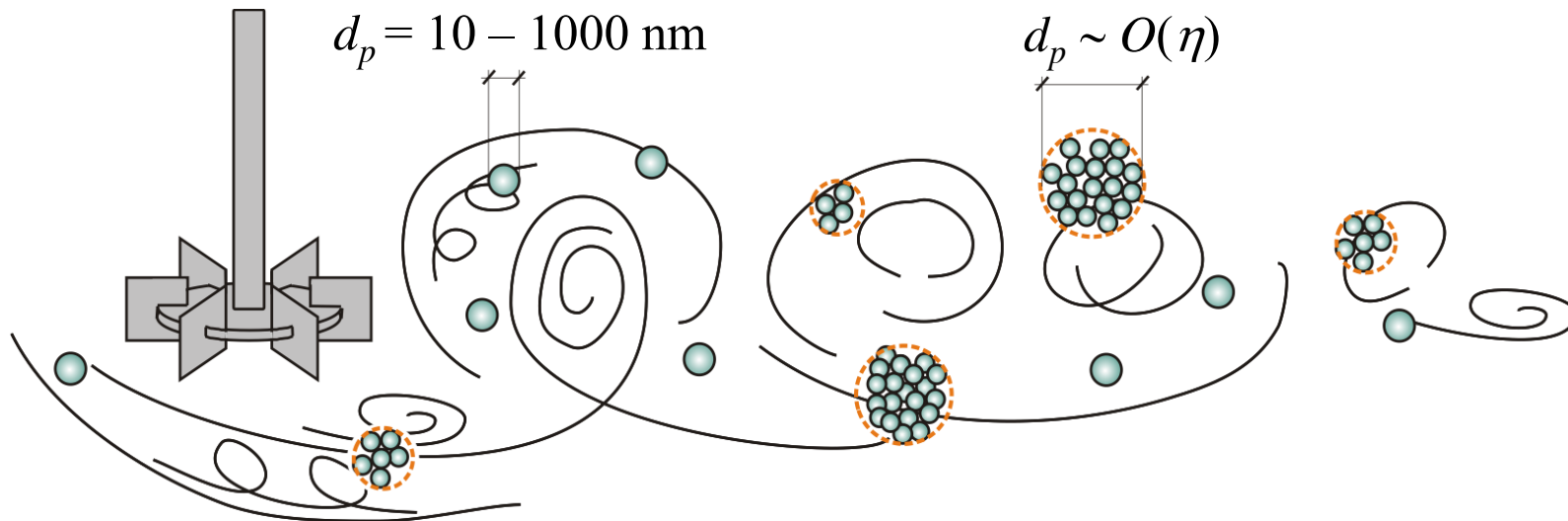
Picture: H. P. Grossart, IGB,  
Leibniz-Institute of Freshwater  
Ecology and Inland Fisheries

# Aggregation of small particles

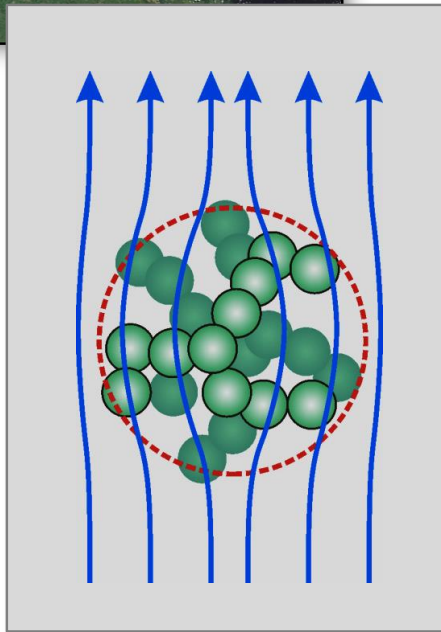


Pictures: M. Soos, D. Marchisio, J. Sefcik, *AIChE J.* (2013), and Soos et al., *J. Colloid Interface Sci.* (2008)

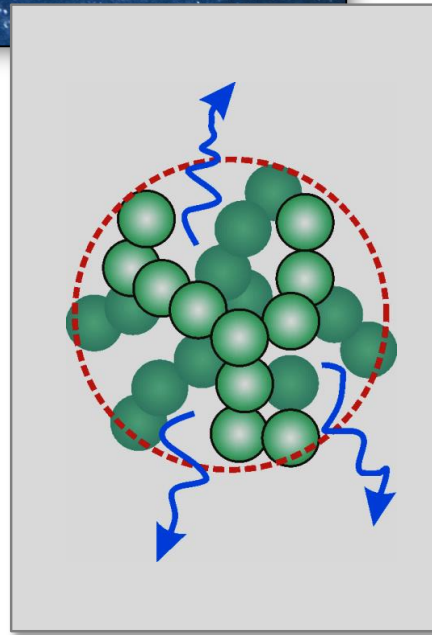
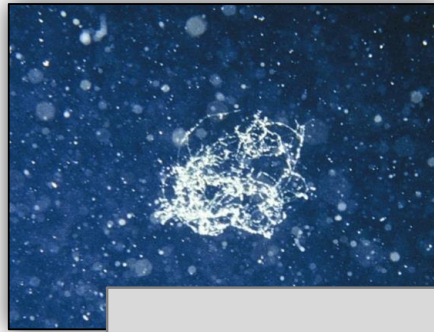
# Aggregation of small particles



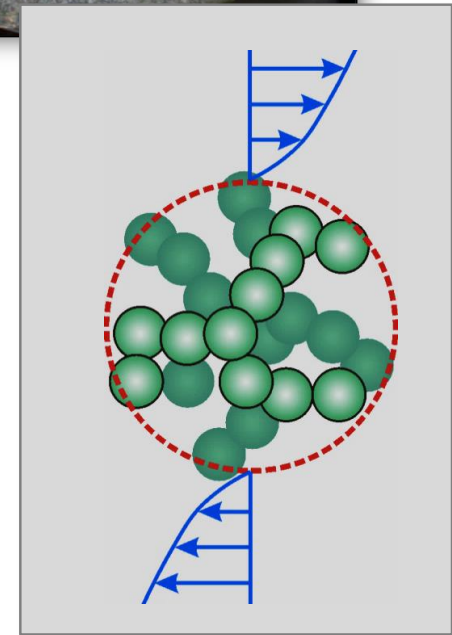
# Structure and restructuring



Hydrodynamic drag  
and settling velocity



Solute loading  
(drugs, nutrients)



Mechanical  
strength





ROYAL INSTITUTE  
OF TECHNOLOGY

# Outline

- Introduction
- Structure of aggregates: aggregate fractal dimension
- Strategy for exploring restructuring
- Population balance model
- Breakup model
- Results
- Conclusions

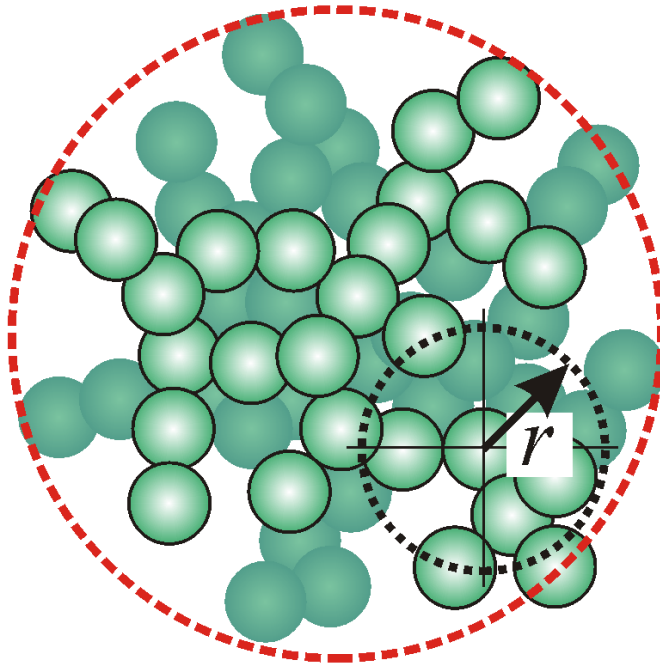


ROYAL INSTITUTE  
OF TECHNOLOGY

# Outline

- Introduction
- **Structure of aggregates: aggregate fractal dimension**
- Strategy for exploring restructuring
- Population balance model
- Breakup model
- Results
- Conclusions

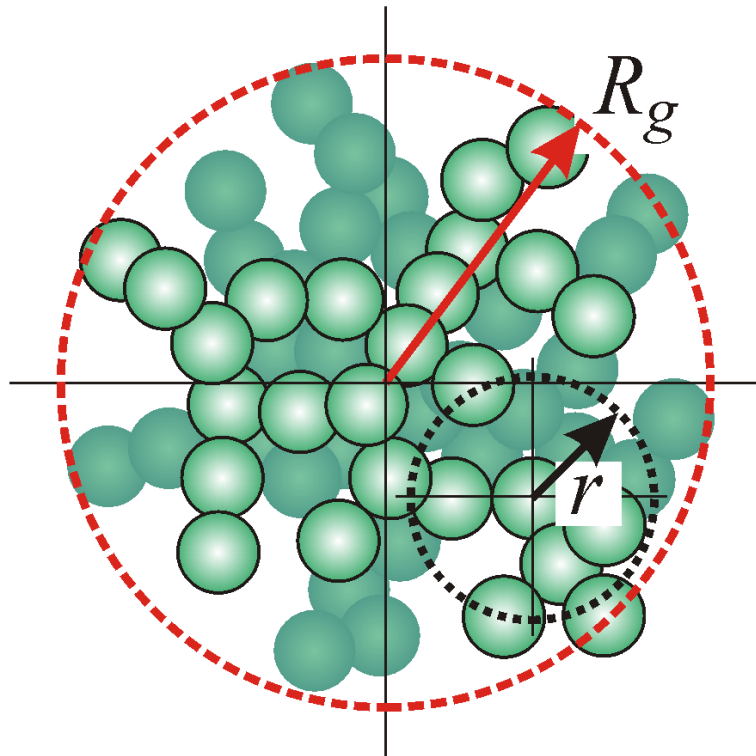
# Aggregate fractal dimension



- Pair-correlation function

$$g(r) = \frac{\langle \rho(x) \rho(x+r) \rangle}{\langle \rho^2(x) \rangle} \sim r^{d_f-3}$$

# Aggregate fractal dimension



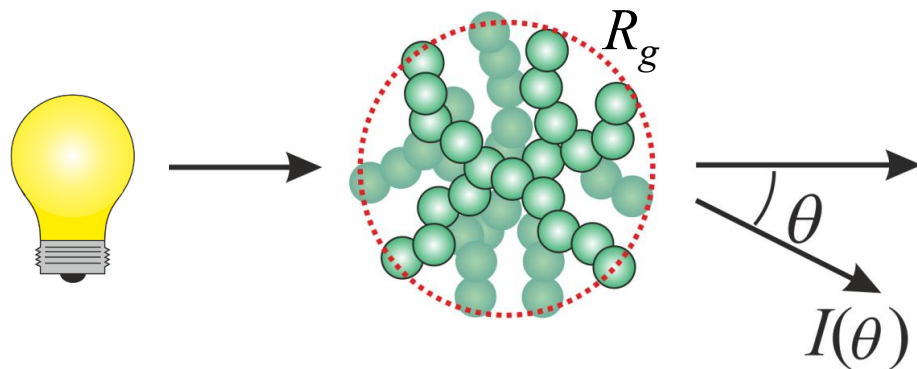
- Pair-correlation function

$$g(r) = \frac{\langle \rho(x)\rho(x+r) \rangle}{\langle \rho^2(x) \rangle} \sim r^{d_f-3}$$

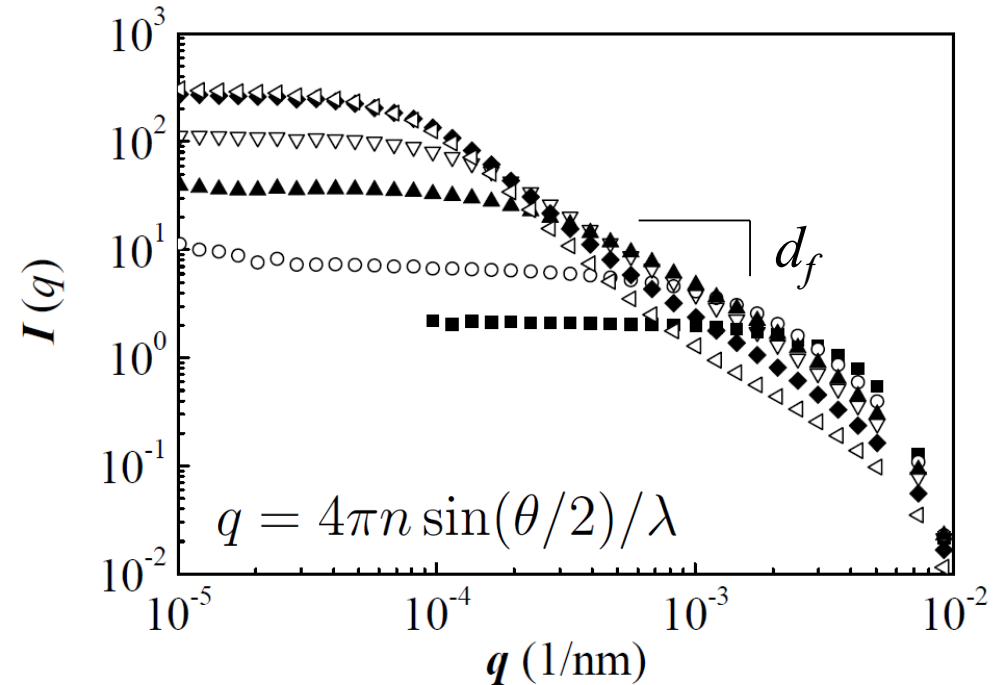
- Scaling of aggregate size

$$i \sim R_g^{d_f}$$

# Measuring mass fractal dimension



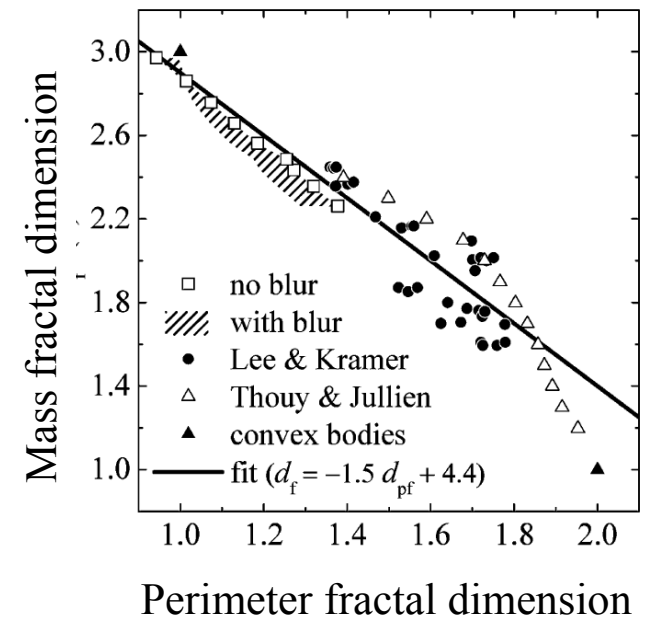
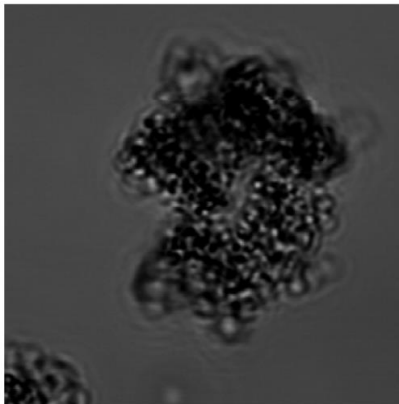
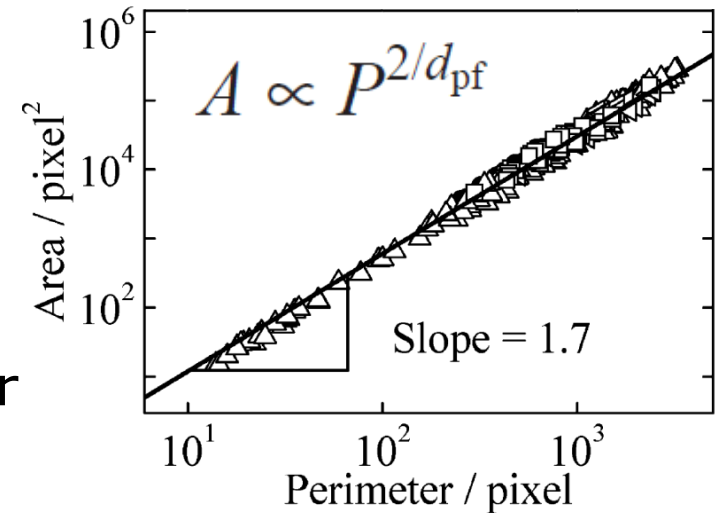
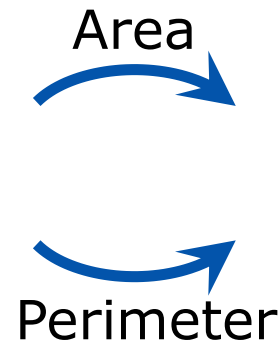
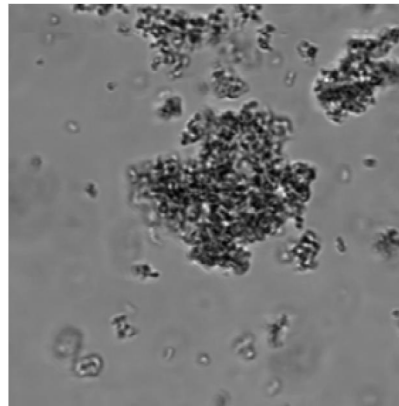
Static Light Scattering



For  $d_p \ll \lambda$  and not too dense aggregates:  
 $d_f$  from the log-log slope

Moussa *et al.*, *Langmuir* (2007)

# Measuring mass fractal dimension



Left: A. S. Moussa, *PhD Thesis, ETH Zurich* (2008)

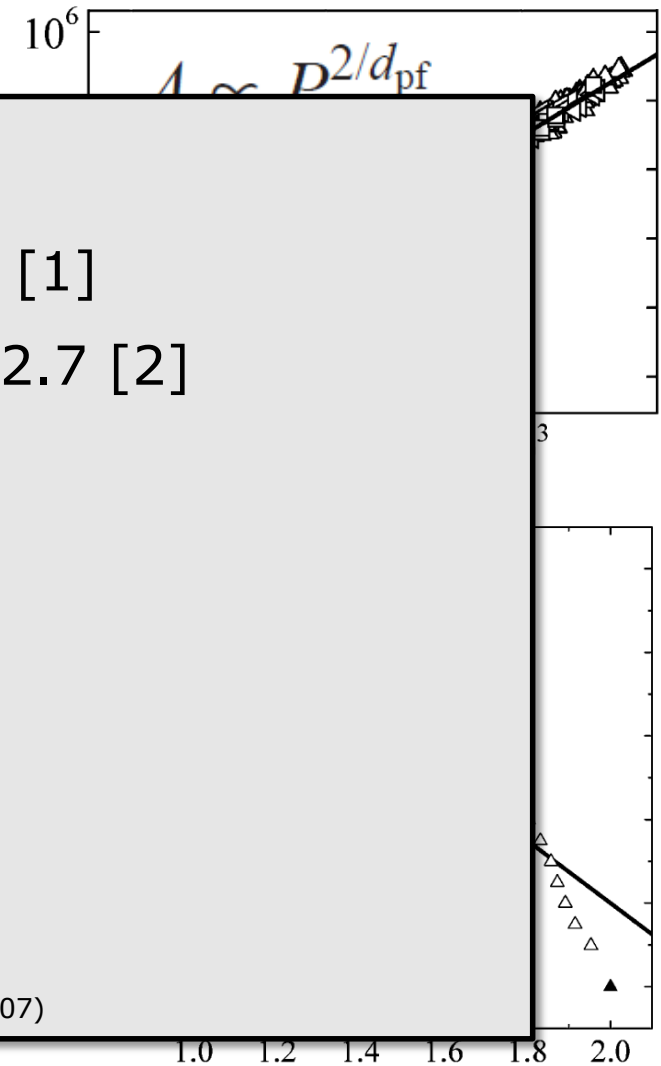
Upper right: Ehrl et al., *Langmuir* (2008), Lower right: Ehrl et al., *J. Phys. Chem. B* (2009)

# Measuring mass fractal dimension

## In (turbulently) stirred fluids

- Early stages of aggregation  $d_f \approx 2.1$  [1]
- Late stages of aggregation  $d_f \approx 2.3-2.7$  [2]

[1] Babler, et al., Langmuir (2010), [2] Soos et al., J. Colloid Interface Sci. (2007)



Left: A. S. Moussa, *PhD Thesis, ETH Zurich* (2008)

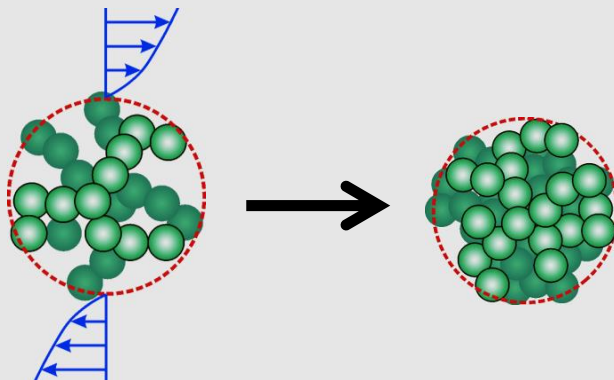
Upper right: Ehrl et al., *Langmuir* (2008), Lower right: Ehrl et al., *J. Phys. Chem. B* (2009)

Perimeter fractal dimension

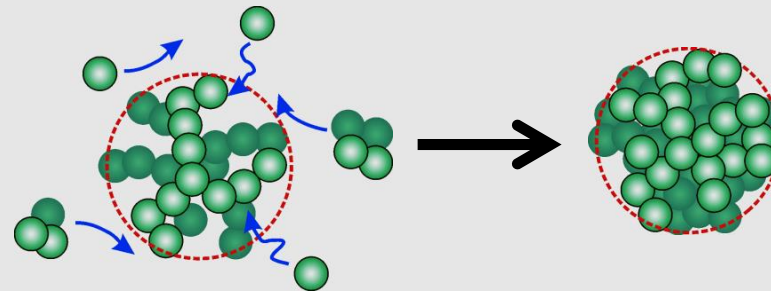
# Measuring mass fractal dimension

## In (turbulently) stirred fluids

- Early stages of aggregation  $d_f \approx 2.1$  [1]
- Late stages of aggregation  $d_f \approx 2.3-2.7$  [2]

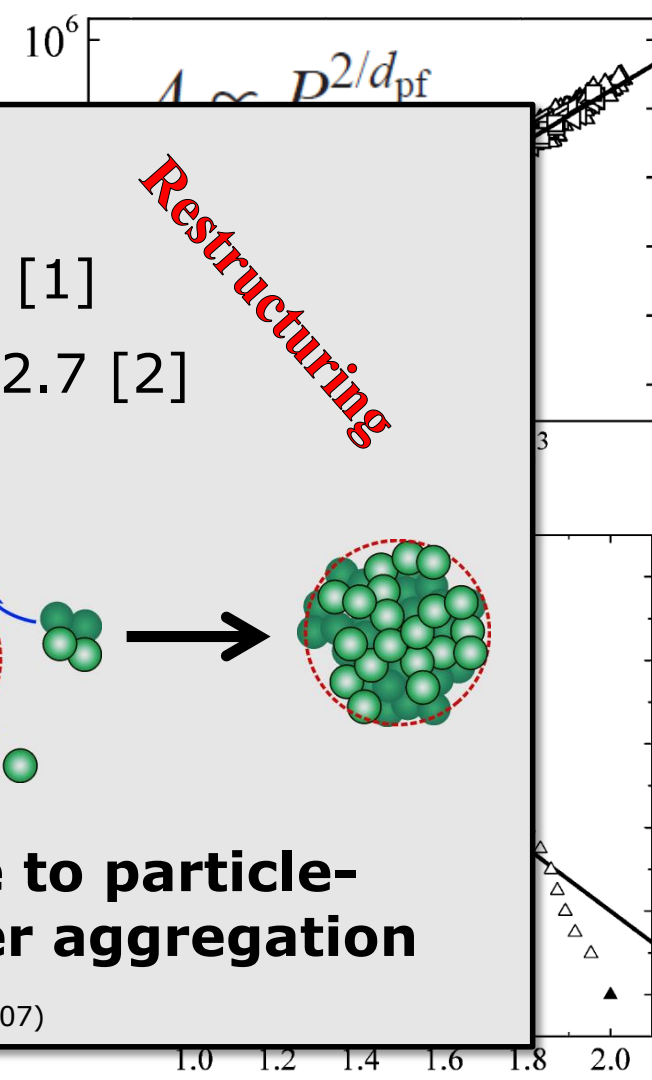


**Due to hydrodynamic stress**



**Due to particle-cluster aggregation**

**Restructuring**



[1] Babler, et al., Langmuir (2010), [2] Soos et al., J. Colloid Interface Sci. (2007)

Left: A. S. Moussa, PhD Thesis, ETH Zurich (2008)

Upper right: Ehrl et al., Langmuir (2008), Lower right: Ehrl et al., J. Phys. Chem. B (2009)

Perimeter fractal dimension



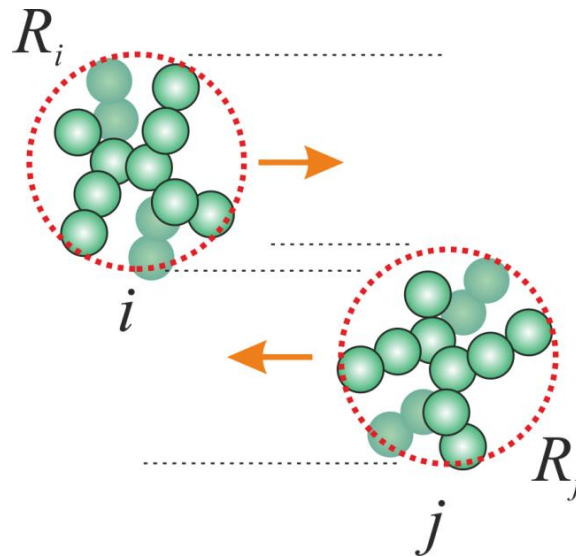
# Alternative approach

Consider an aggregate  $i$  colliding with an aggregate  $j$

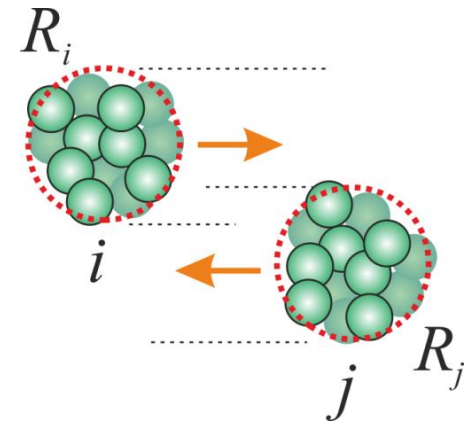
$i, j$  = number of primary particle per aggregate

$$i \sim R_g^{d_f}$$

Open aggregate  
Small  $d_f$



Dense aggregates  
large  $d_f$



$$K_{ij} \sim (R_i + R_j)^3 \sim (i^{1/d_f} + j^{1/d_f})^3$$

$\Rightarrow d_f$  can be estimated from measuring the aggregation rate



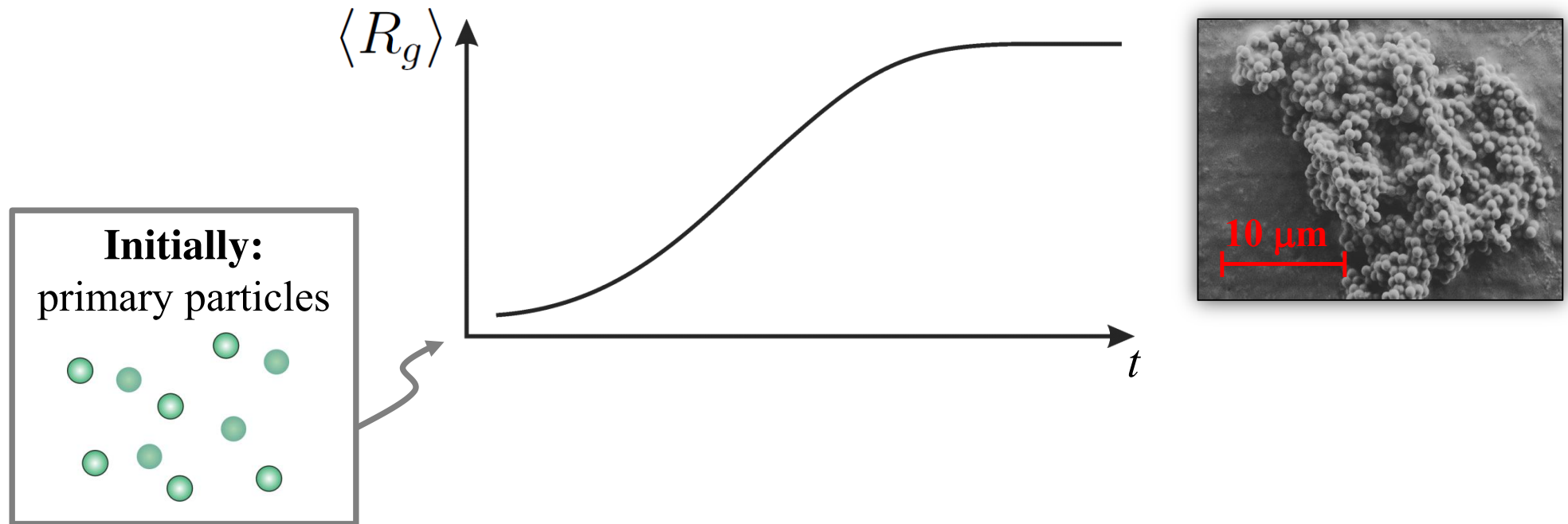
ROYAL INSTITUTE  
OF TECHNOLOGY

# Outline

- Introduction
- Structure of aggregates: aggregate fractal dimension
- **Strategy for exploring restructuring**
- Population balance model
- Breakup model
- Results
- Conclusions

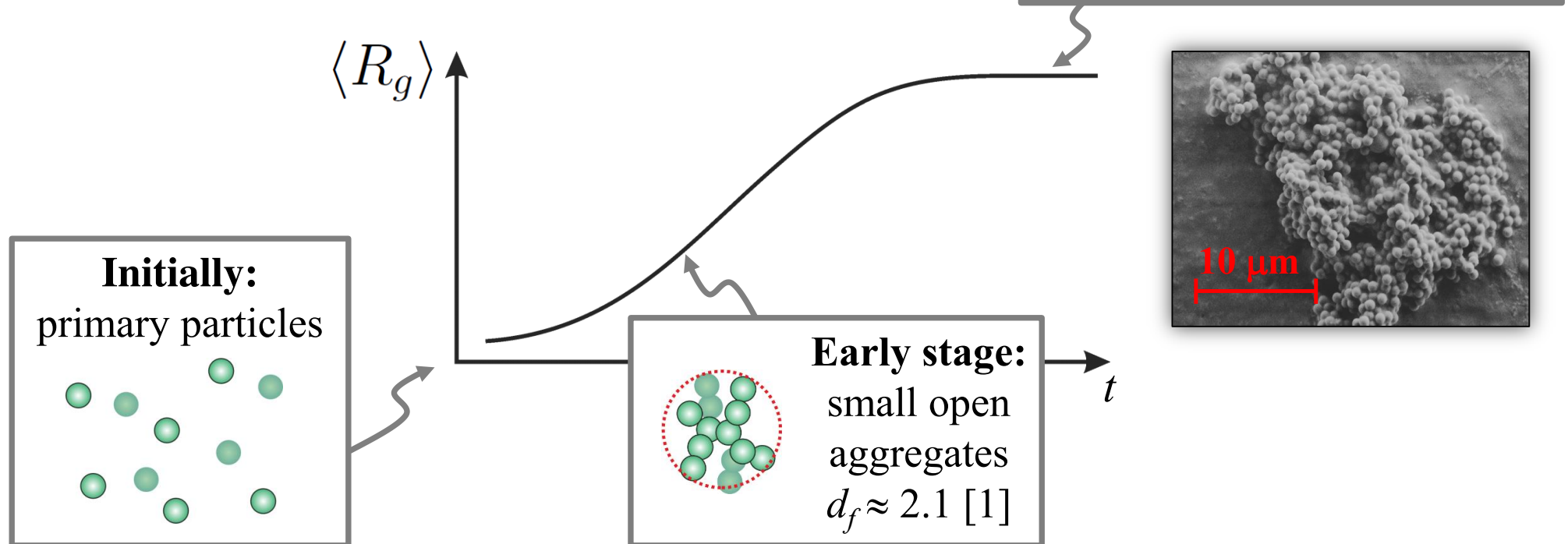
# Strategy pursued

## Exploring restructuring during aggregation of primary particles



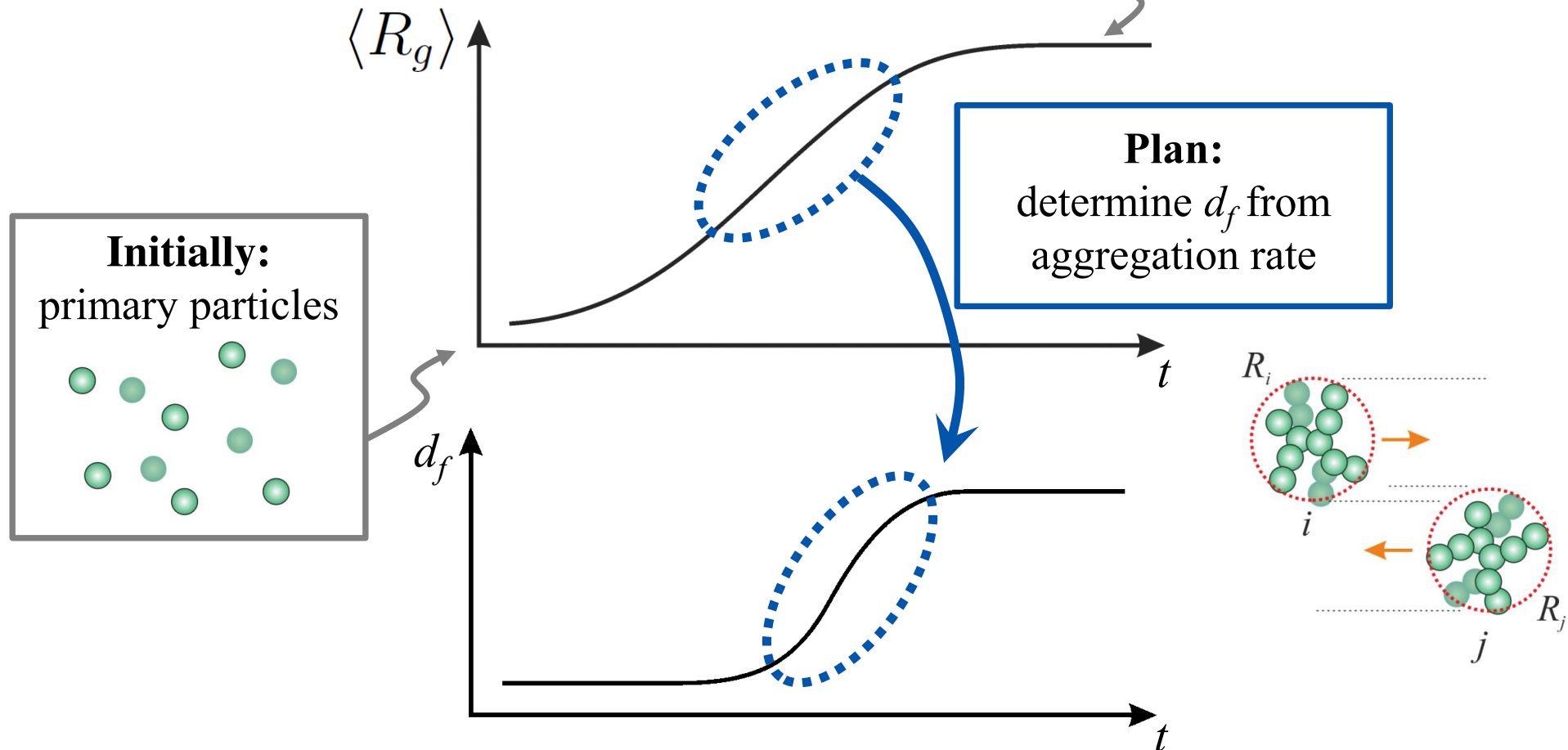
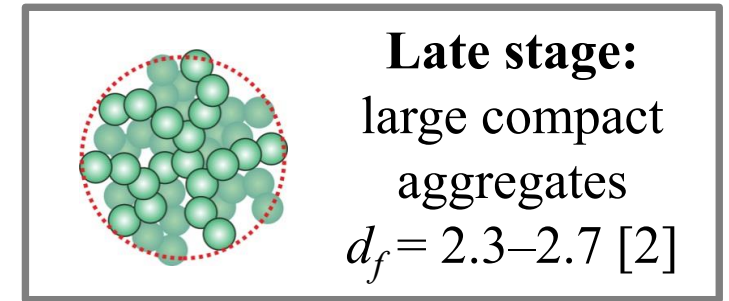
# Strategy pursued

## Exploring restructuring during aggregation of primary particles

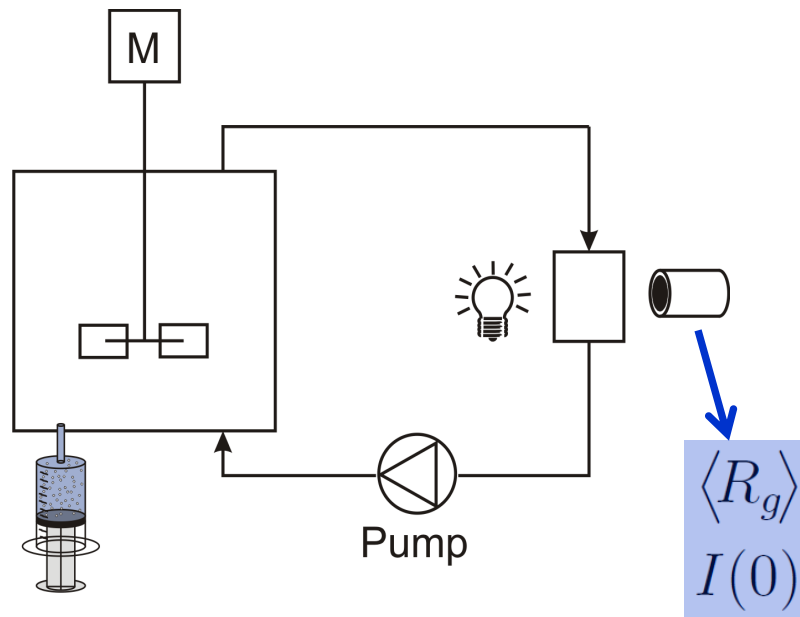


# Strategy pursued

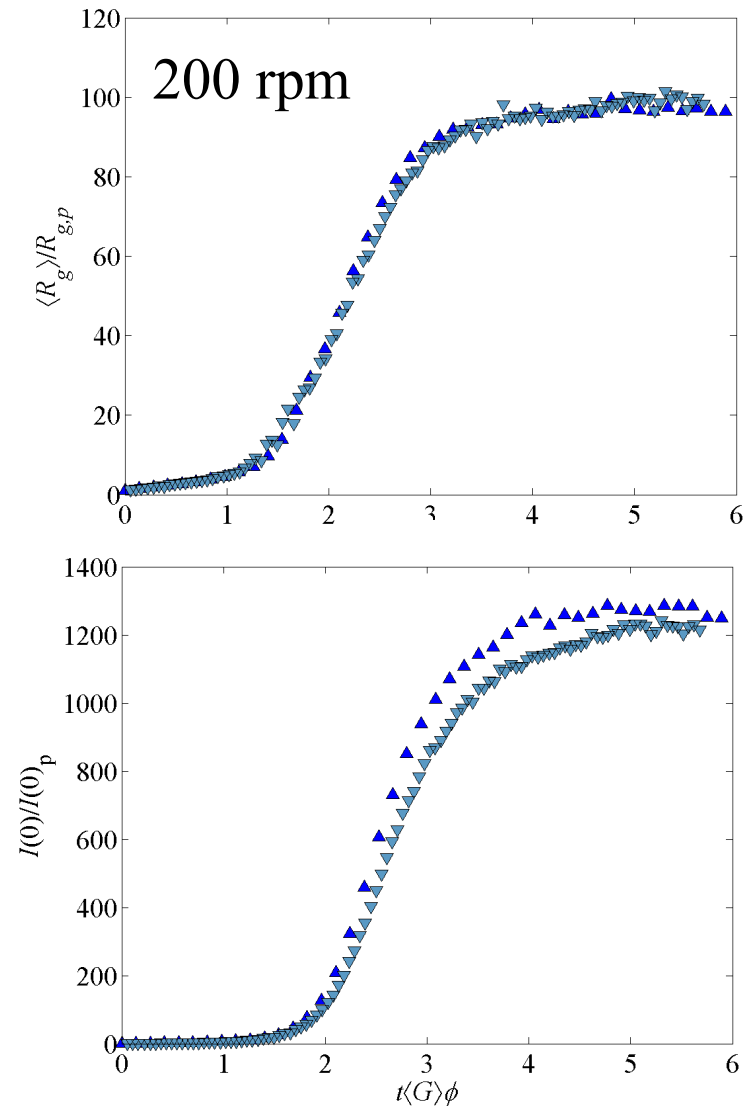
**Exploring restructuring during aggregation of primary particles**



# Experimental

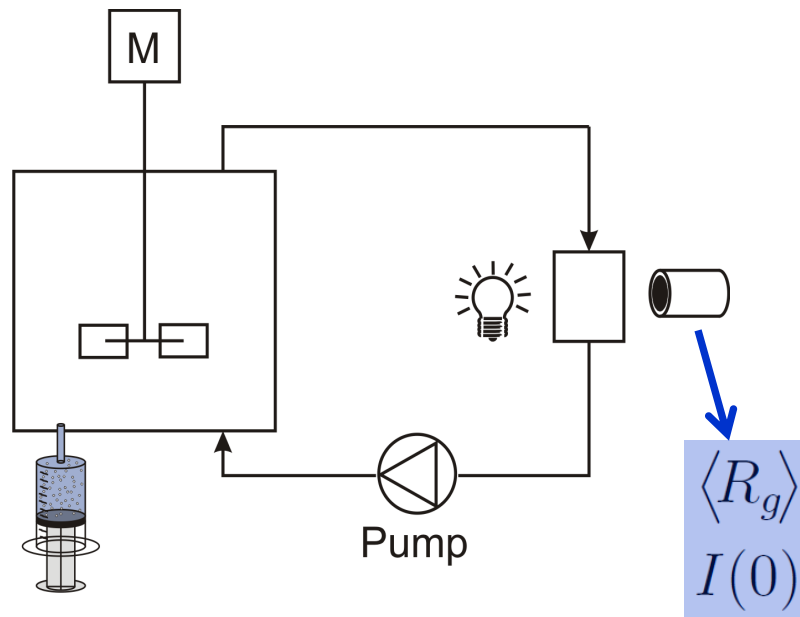


- Polystyrene particles
- $d_p = 420 \text{ nm}$ ,  $\phi = 2 \times 10^{-5}$
- Coagulant:  $\text{Al}(\text{NO}_3)_3$ , 0.16 w%
- Fully destabilized particles

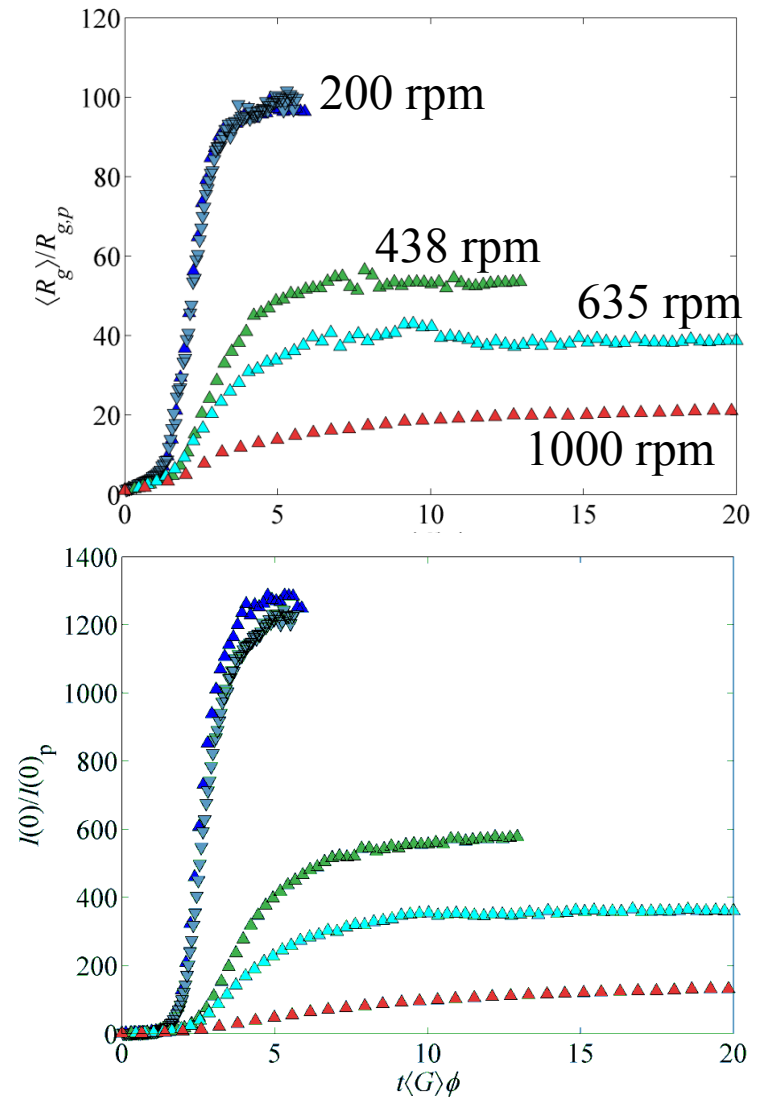


Ehrl et al., *Langmuir* (2009)

# Experimental



- Polystyrene particles
- $d_p = 420 \text{ nm}$ ,  $\phi = 2 \times 10^{-5}$
- Coagulant:  $\text{Al}(\text{NO}_3)_3$ , 0.16 w%
- Fully destabilized particles



Ehrl et al., *Langmuir* (2009)

# Outline

- Introduction
- Structure of aggregates: aggregate fractal dimension
- Strategy for exploring restructuring
- **Population balance model**
- Breakup model
- Results
- Conclusions





ROYAL INSTITUTE  
OF TECHNOLOGY

# Population balance model

$$\begin{aligned} \frac{dN_i}{dt} = & \frac{1}{2} \sum_{j=1}^{i-1} K_{A,j,i-j} N_j N_{i-j} - N_i \sum_{j=1}^{\infty} K_{A,i,j} N_j \\ & - K_{B,i} N_i + \sum_{j=i+1}^{\infty} g_{i,j} K_{B,j} N_j \end{aligned}$$



ROYAL INSTITUTE  
OF TECHNOLOGY

# Population balance model

$$\frac{dN_i}{dt} = \frac{1}{2} \sum_{j=1}^{i-1} K_{A,j,i-j} N_j N_{i-j} - N_i \sum_{j=1}^{\infty} K_{A,i,j} N_j - K_{B,i} N_i + \sum_{j=i+1}^{\infty} g_{i,j} K_{B,j} N_j$$

$d_f(t)$  (with an arrow pointing to the aggregation term)

- **Aggregation**  $\longrightarrow$  Saffman&Turner type [1]
- **Breakup**  $\longrightarrow$  Critical stress model [3]  
Binary breakup
- **Evolving  $d_f$**   $\longrightarrow$  Pre-described function

[1] Babler, AIChE J. (2008), [2] Babler et al., Langmuir (2010), [3] Babler et al. J. Fluid Mech. (2008), Babler et al. PRE (2012)

# Population balance model

$$\frac{dN_i}{dt} = \frac{1}{2} \sum_{j=1}^{i-1} K_{A,j,i-j} N_j N_{i-j} - N_i \sum_{j=1}^{\infty} K_{A,i,j} N_j - K_{B,i} N_i + \sum_{j=i+1}^{\infty} g_{i,j} K_{B,j} N_j$$

$d_f(t)$

- **Aggregation**  $\longrightarrow$  Saffman&Turner type [1]
- **Breakup**  $\longrightarrow$  Critical stress model [3]  
Binary breakup
- **Evolving  $d_f$**   $\longrightarrow$  Pre-described function

- Light scattering model [2]

$$\frac{\langle R_g \rangle}{R_{g,p}} = \left( \frac{\sum_{i=1}^{\infty} i^{d_f(2-c)} N_i}{\sum_{i=1}^{\infty} i^{2-c} N_i} \right)^{1/2}$$

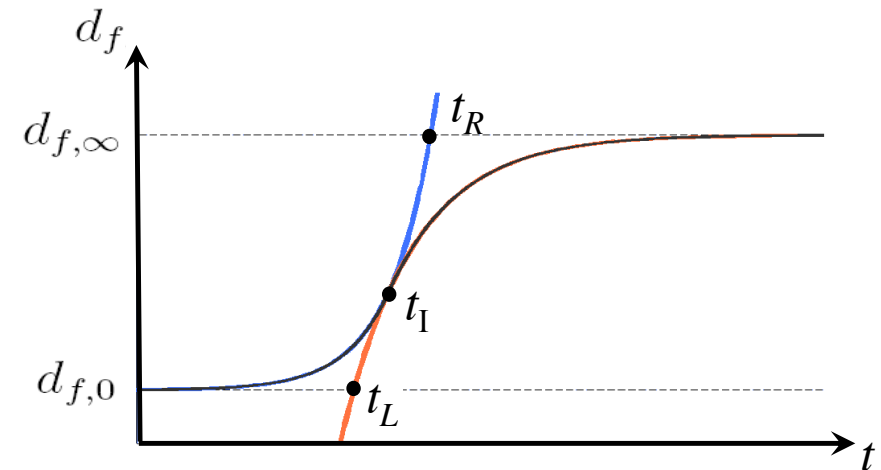
$$\frac{I(0)}{I(0)_p} = \frac{\sum_{i=1}^{\infty} i^{2-c} N_i}{\sum_{i=1}^{\infty} i N_i}$$

$c$  = correction factor for multiple scattering, important for

- large aggregates
- dense aggregates

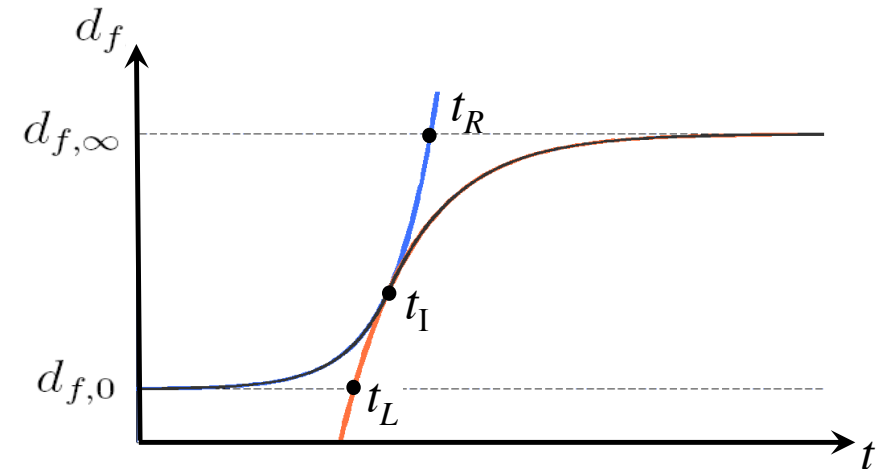
# Evolving fractal dimension

- Pre-described function evolving smoothly between two plateaus

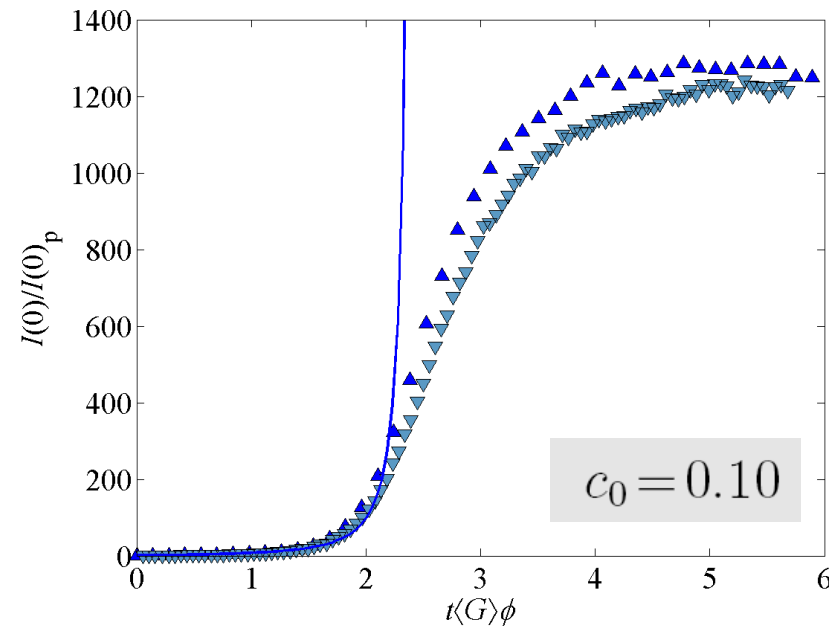
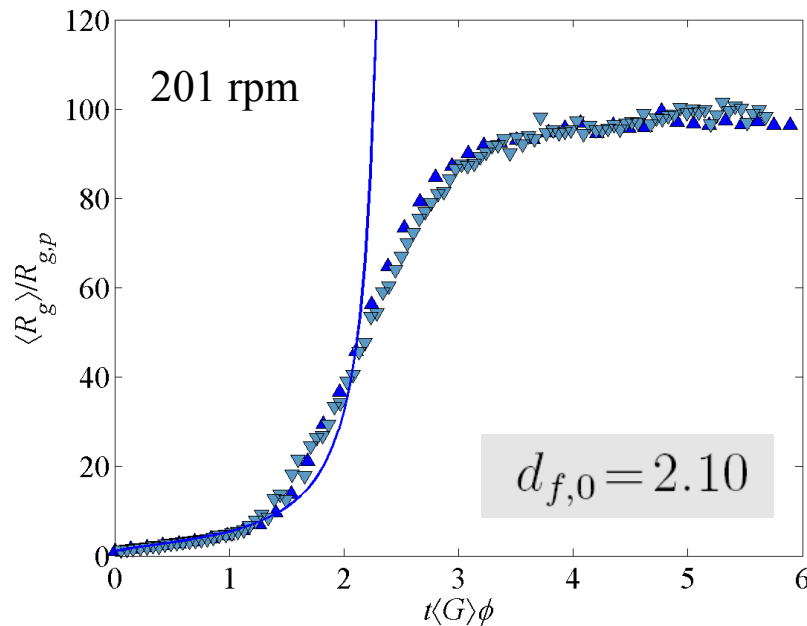


# Evolving fractal dimension

- Pre-described function evolving smoothly between two plateaus



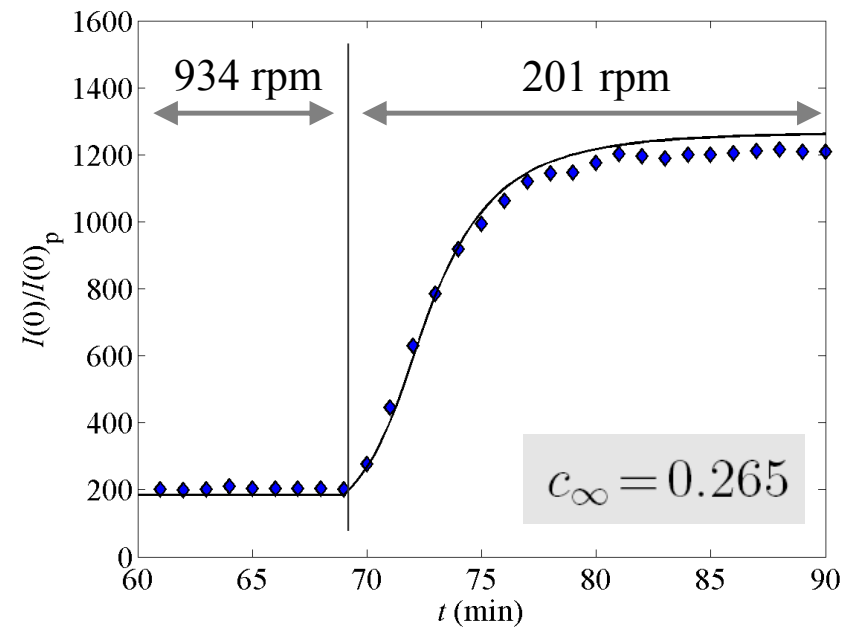
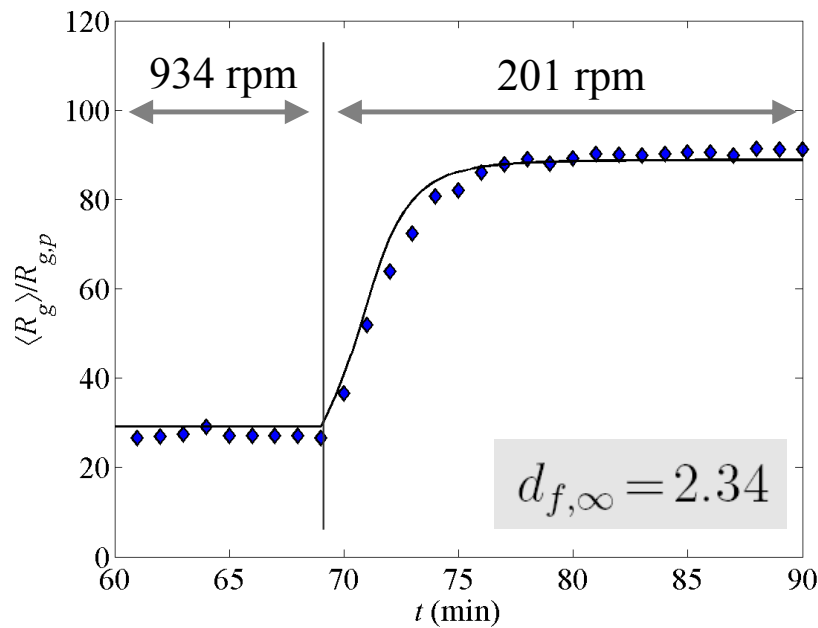
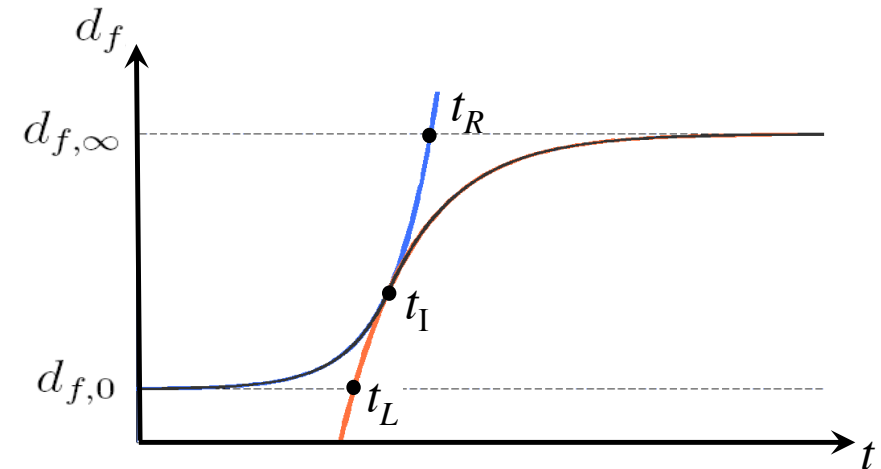
## Initial fractal dimension



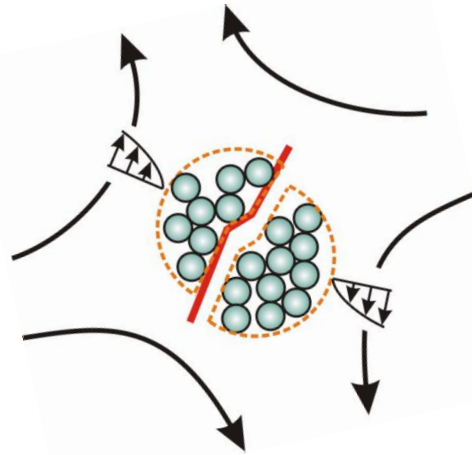
# Evolving fractal dimension

- Pre-described function evolving smoothly between two plateaus

## Final fractal dimension

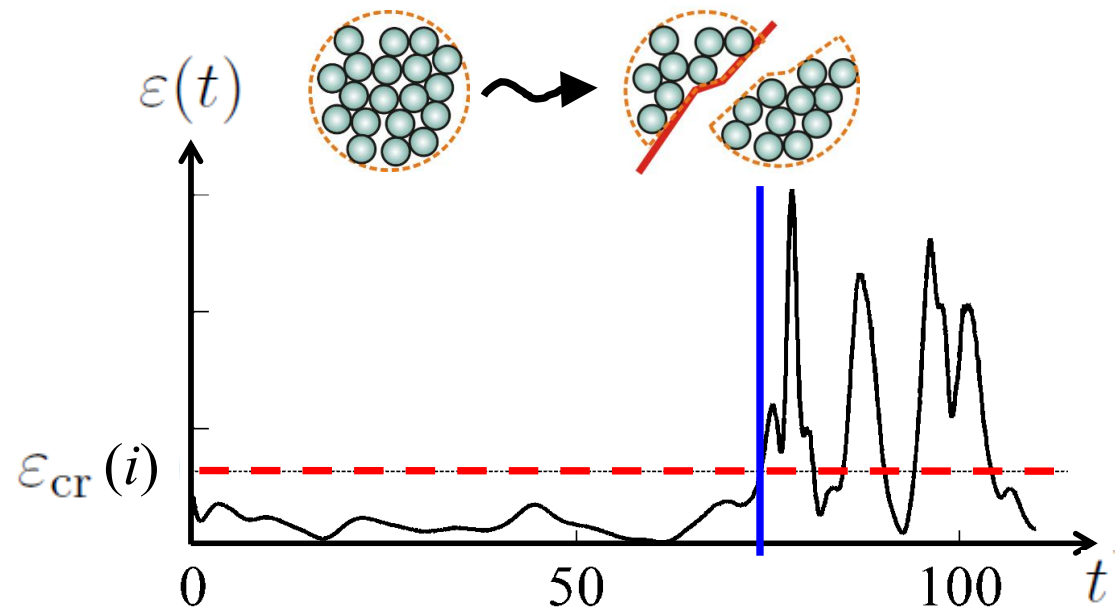


# Aggregate breakup in turbulence



- $\exists$  critical stress  $\sigma_{cr} = \sigma_{cl}(i)$
- Instantaneous breakup
- Small, inertialless particles

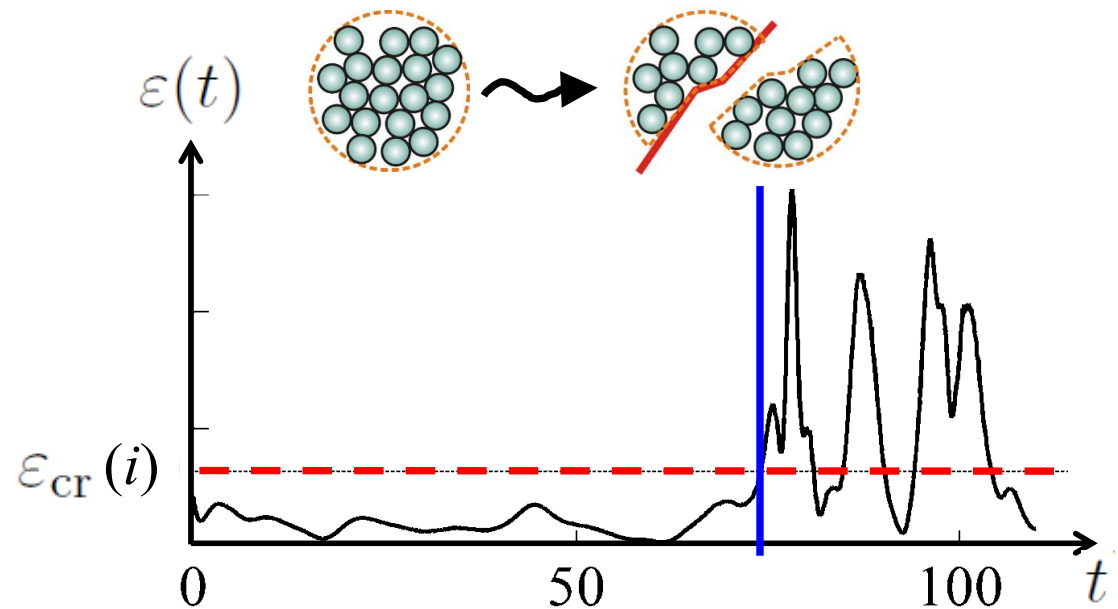
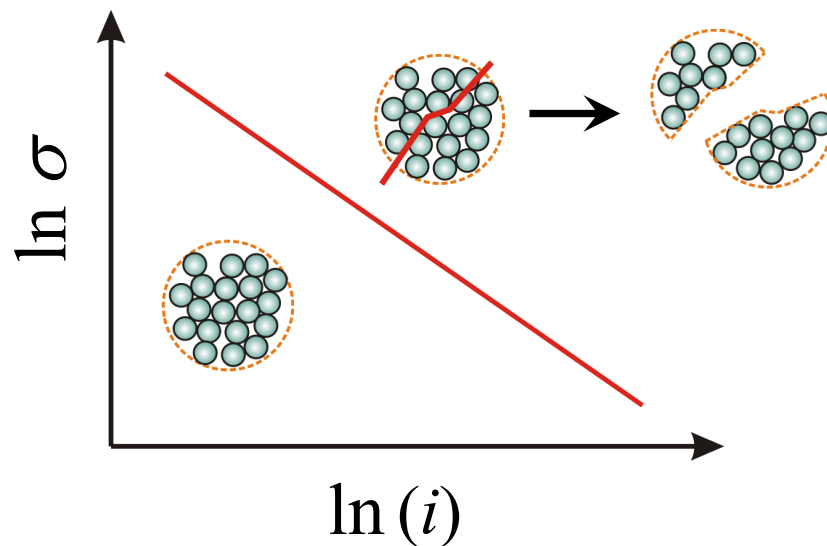
$$\sigma \sim \mu(\nu/\varepsilon)^{1/2}$$



# Aggregate breakup in turbulence

- Critical stress depends on the aggregate properties, i.e., its "size" ( $i$ )
- Power law behavior

$$\sigma_{cr}(i) \sim i^{-m/2}$$



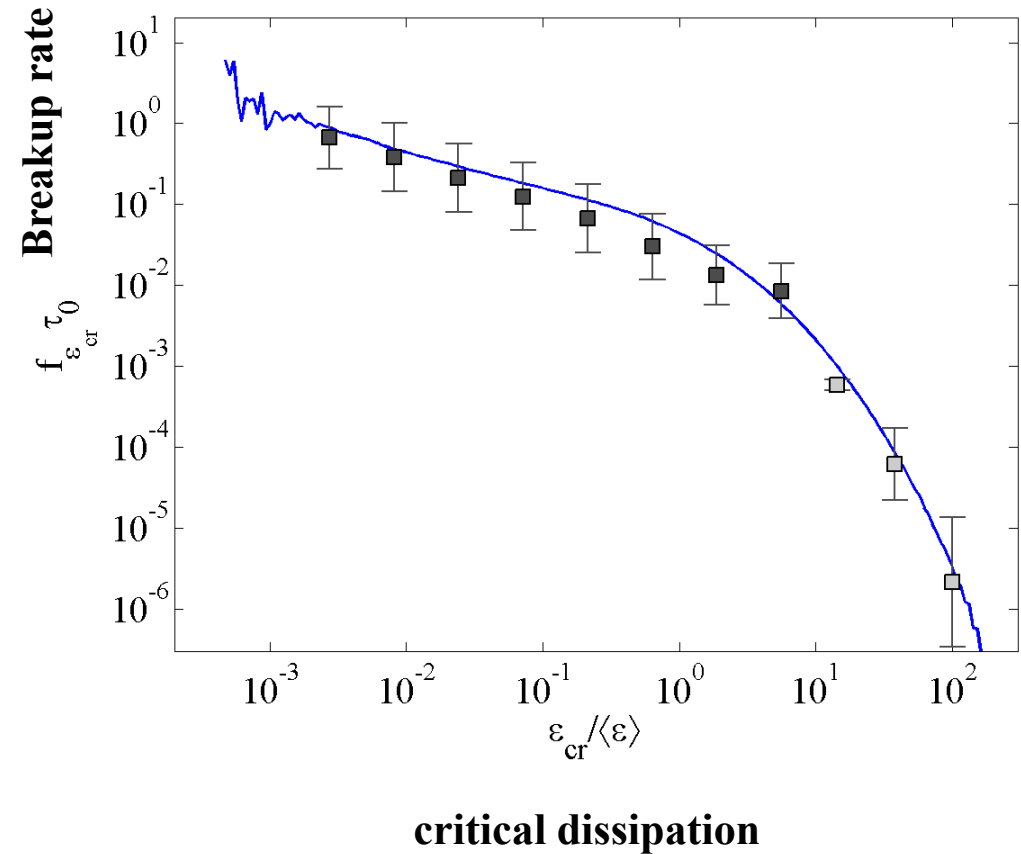
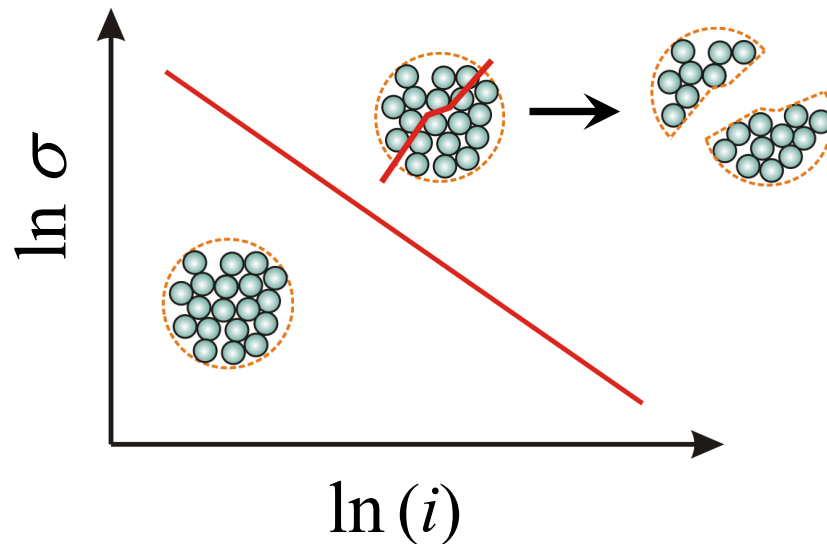
Babler et al. *J. Fluid Mech.* (2008); Babler, Biferale, Lanotte, *PRE* (2012)



# Aggregate breakup in turbulence

- Critical stress depends on the aggregate properties, i.e., its "size" ( $i$ )
- Power law behavior

$$\sigma_{cr}(i) \sim i^{-m/2}$$



Babler et al. *J. Fluid Mech.* (2008); Babler, Biferale, Lanotte, *PRE* (2012)

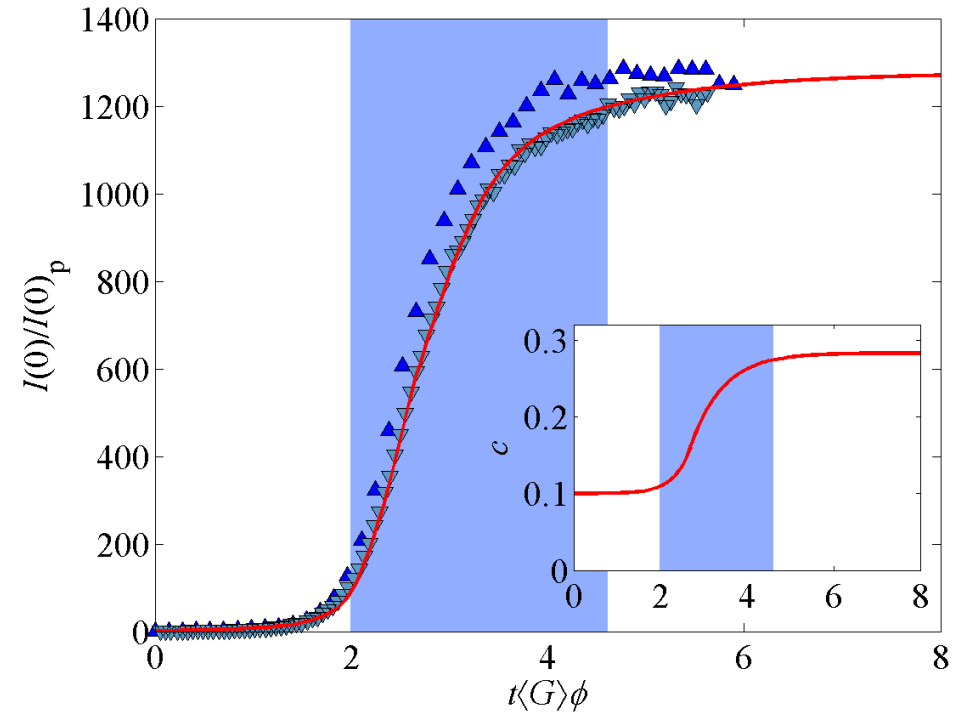
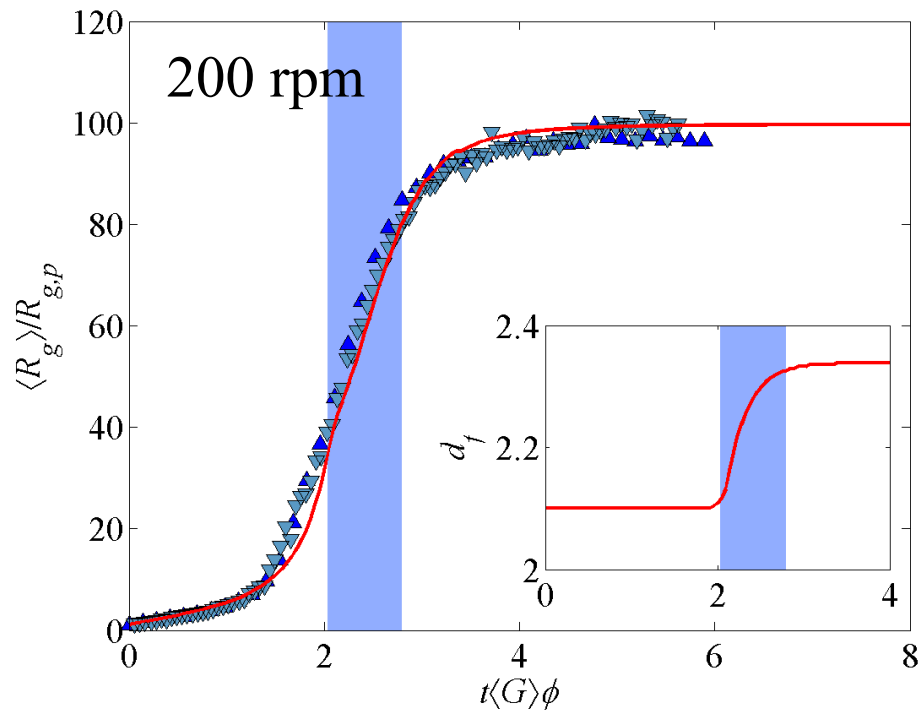


ROYAL INSTITUTE  
OF TECHNOLOGY

# Outline

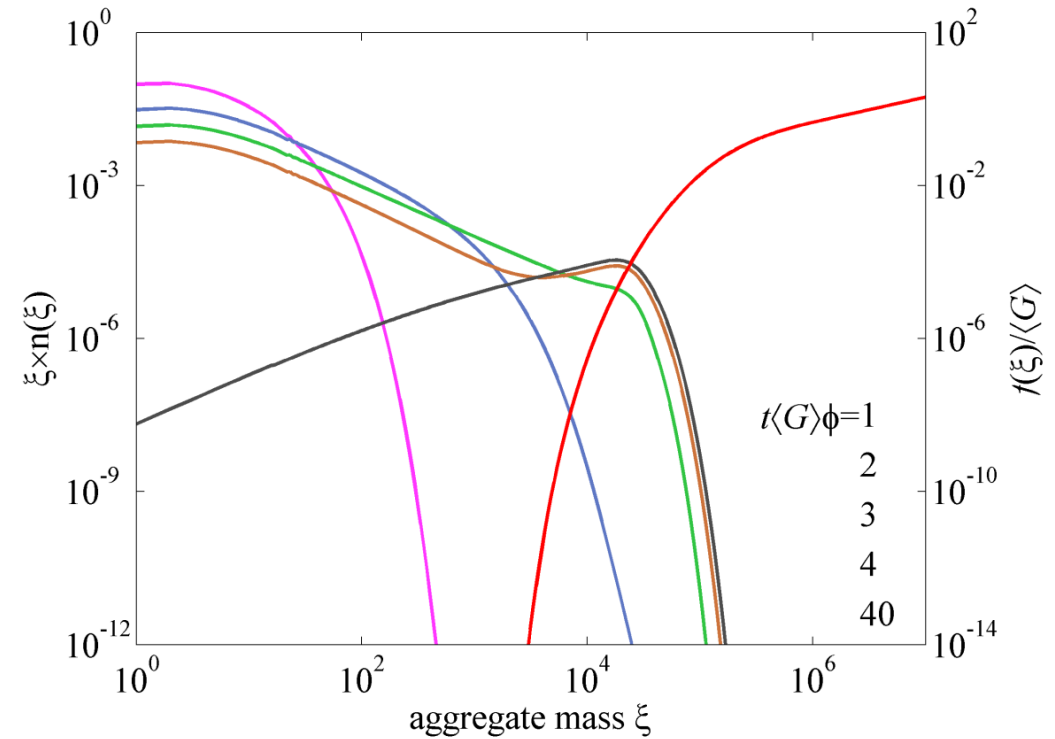
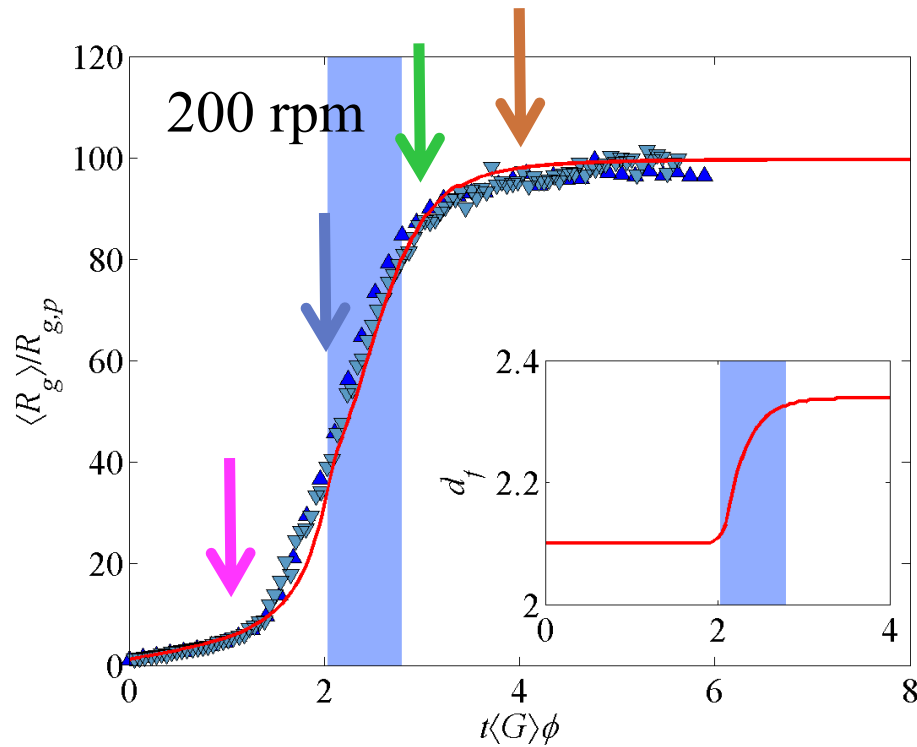
- Introduction
- Structure of aggregates: aggregate fractal dimension
- Strategy for exploring restructuring
- Population balance model
- Breakup model
- **Results**
- Conclusions

# Evolution of fractal dimension



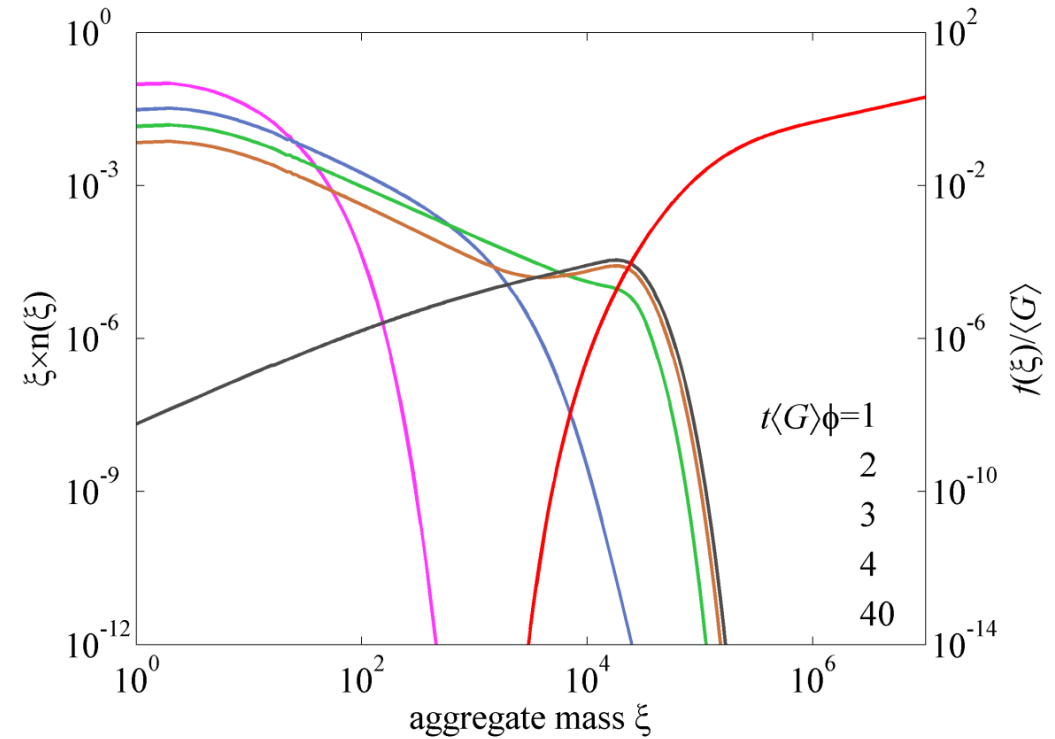
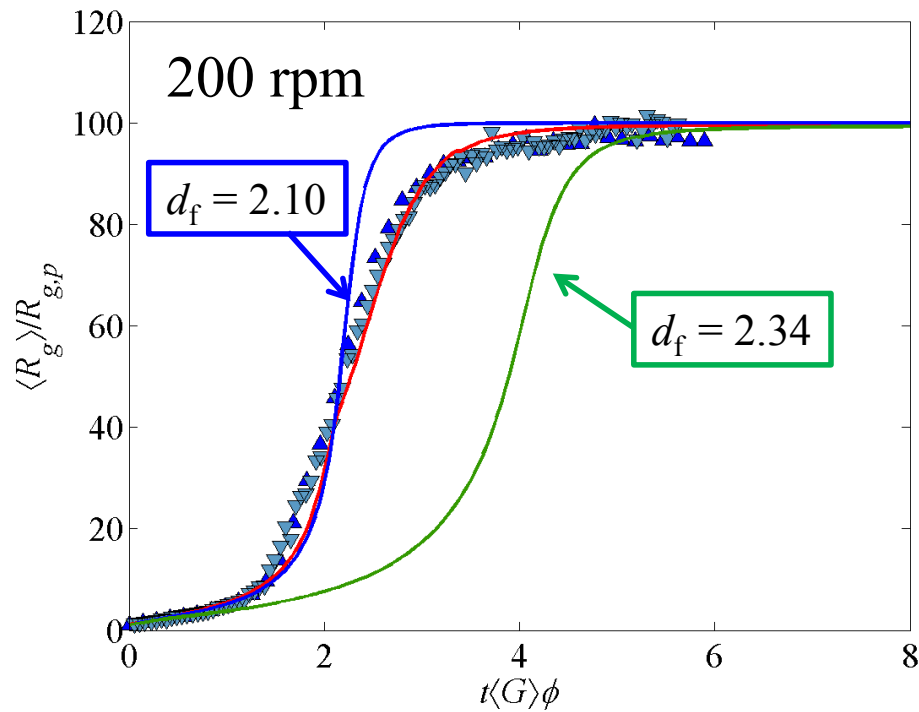
Fitting  $d_f(t)$  and  $c(t)$  to the evolution of  $\langle R_g \rangle$  and  $I(0)$

# Aggregate size distribution



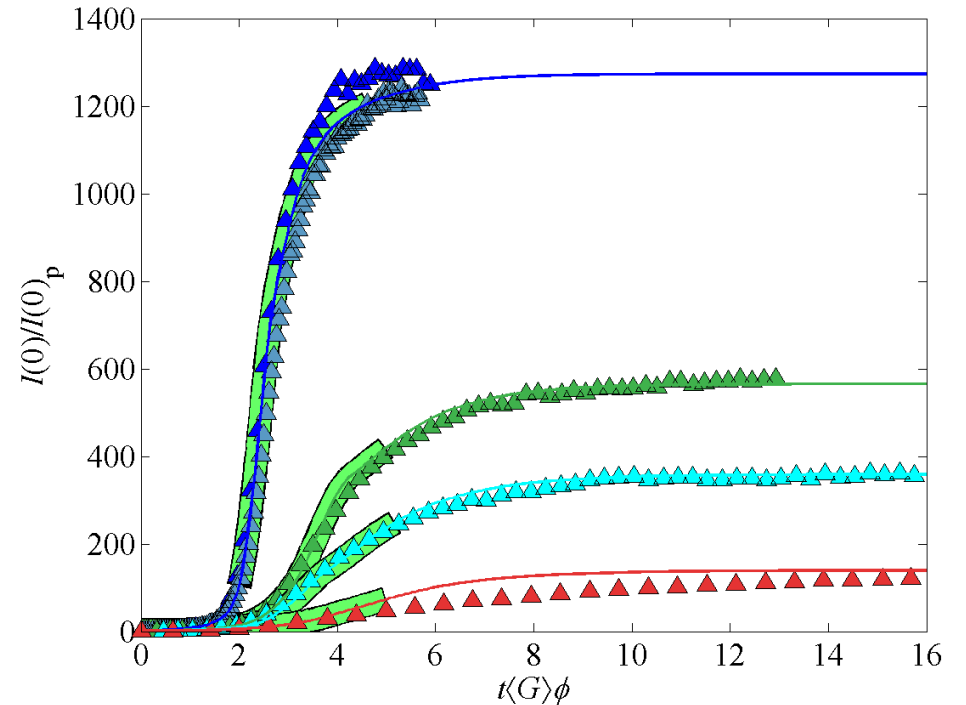
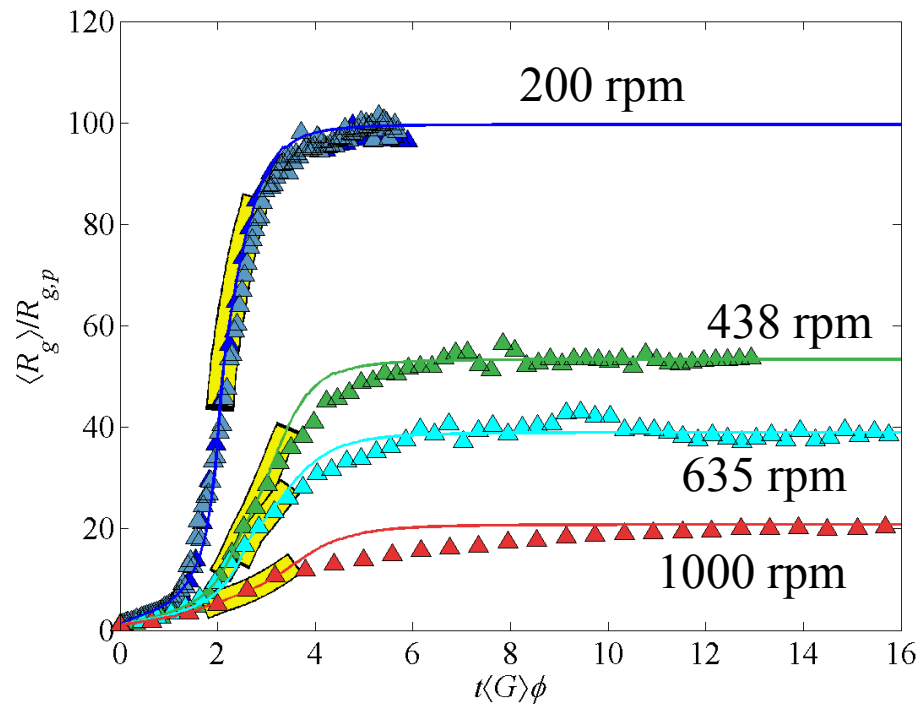
Fitting  $d_f(t)$  and  $c(t)$  to the evolution of  $\langle R_g \rangle$  and  $I(0)$

# Constant fractal dimension



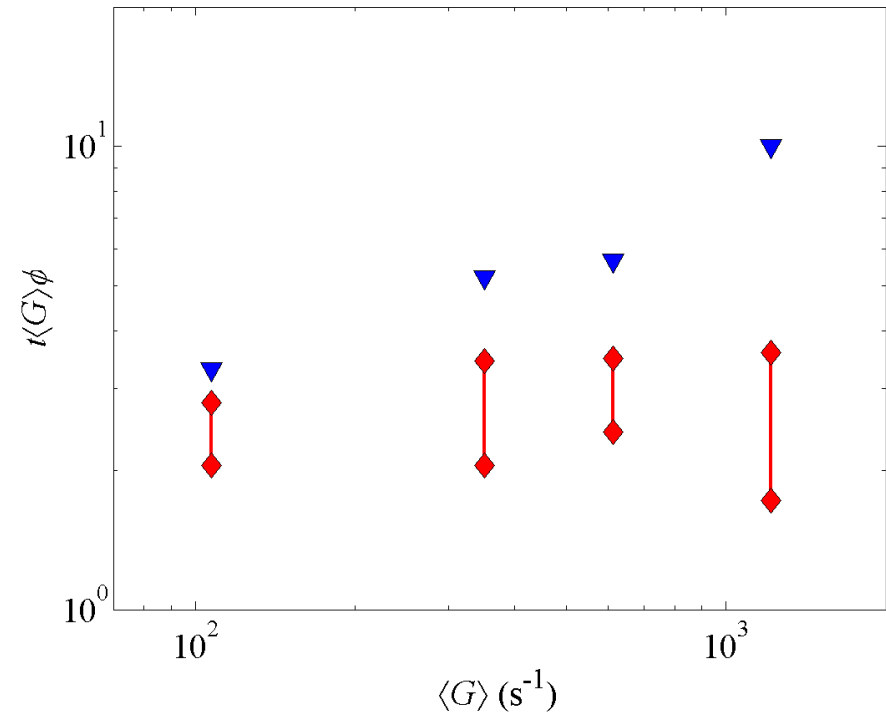
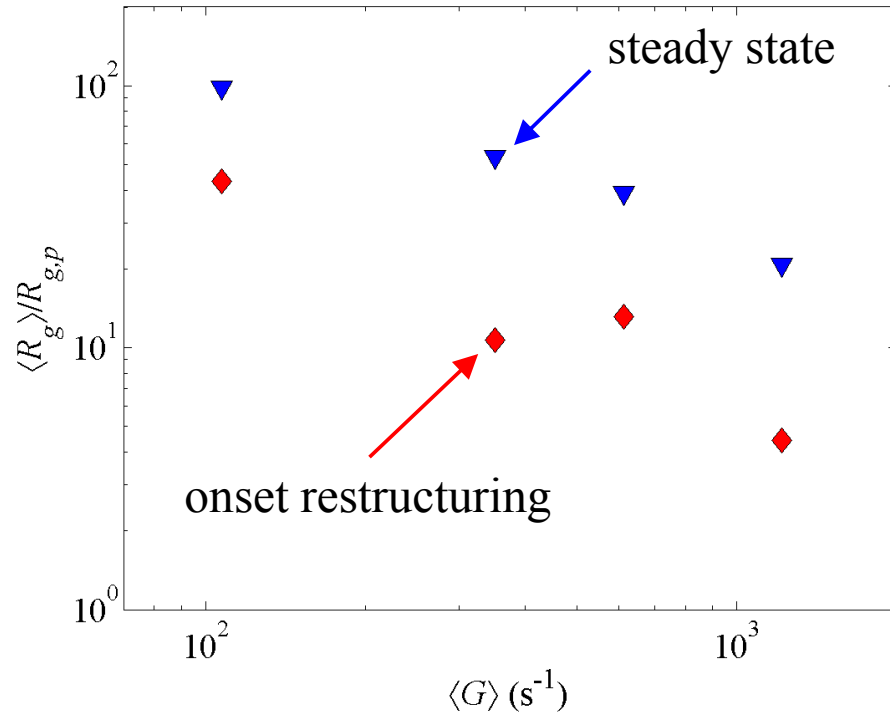
Fitting  $d_f(t)$  and  $c(t)$  to the evolution of  $\langle R_g \rangle$  and  $I(0)$

# Various stirring speeds

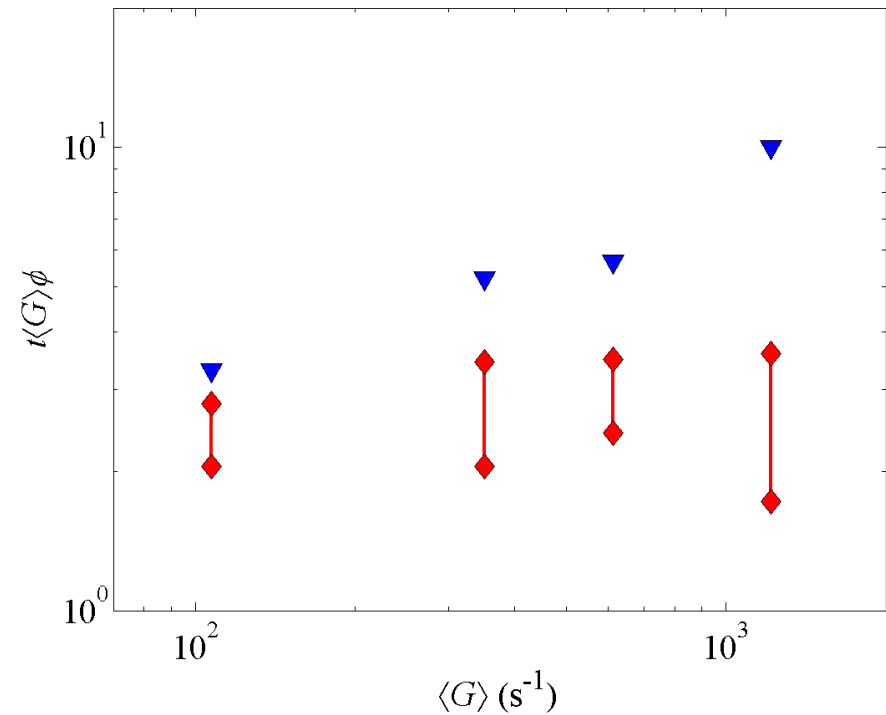
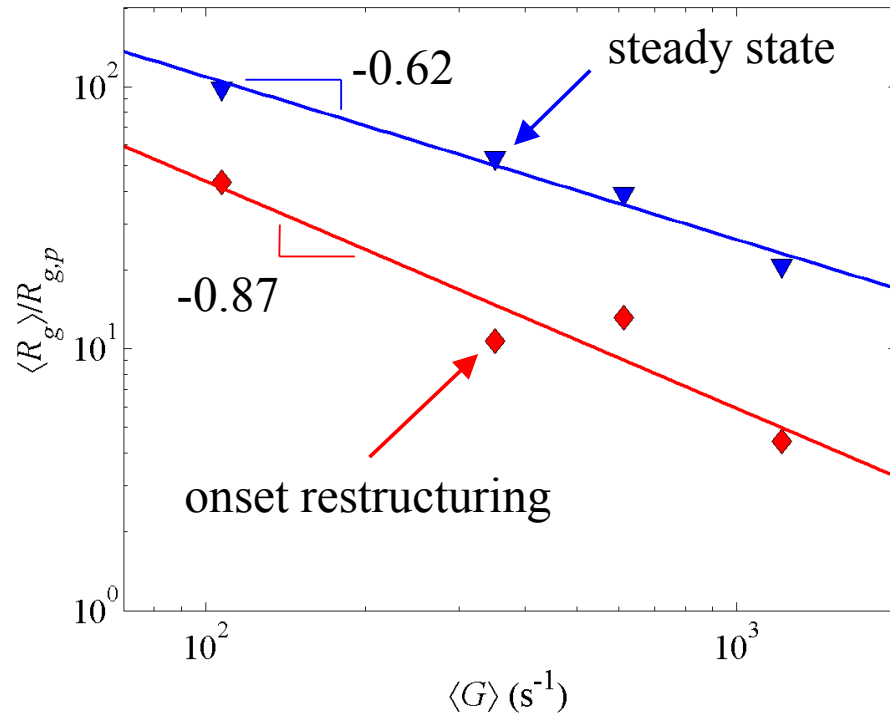


Fitting  $d_f(t)$  and  $c(t)$  to the evolution of  $\langle R_g \rangle$  and  $I(0)$

# Onset and duration of restructuring



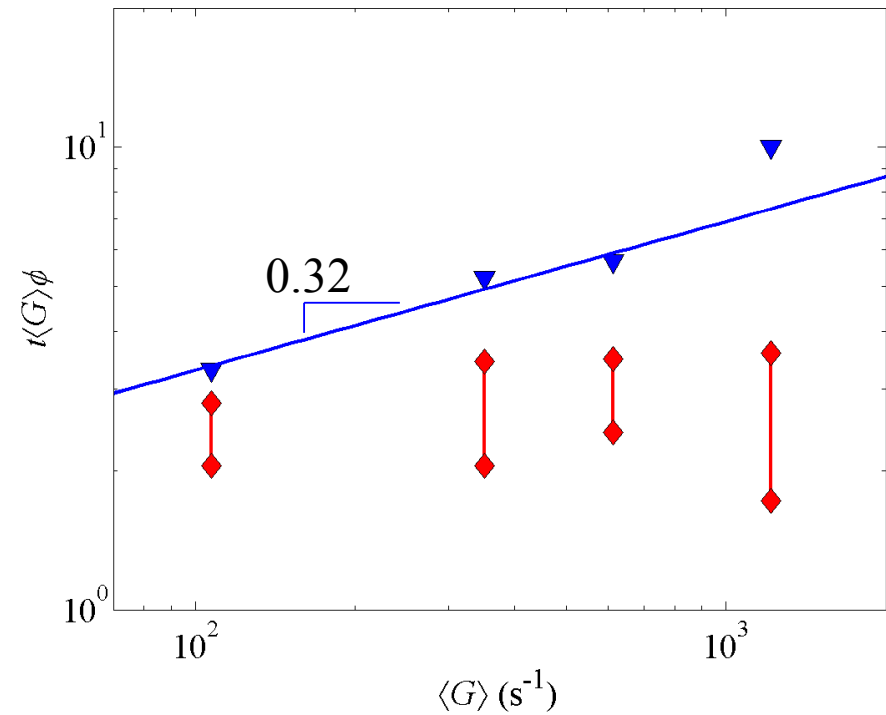
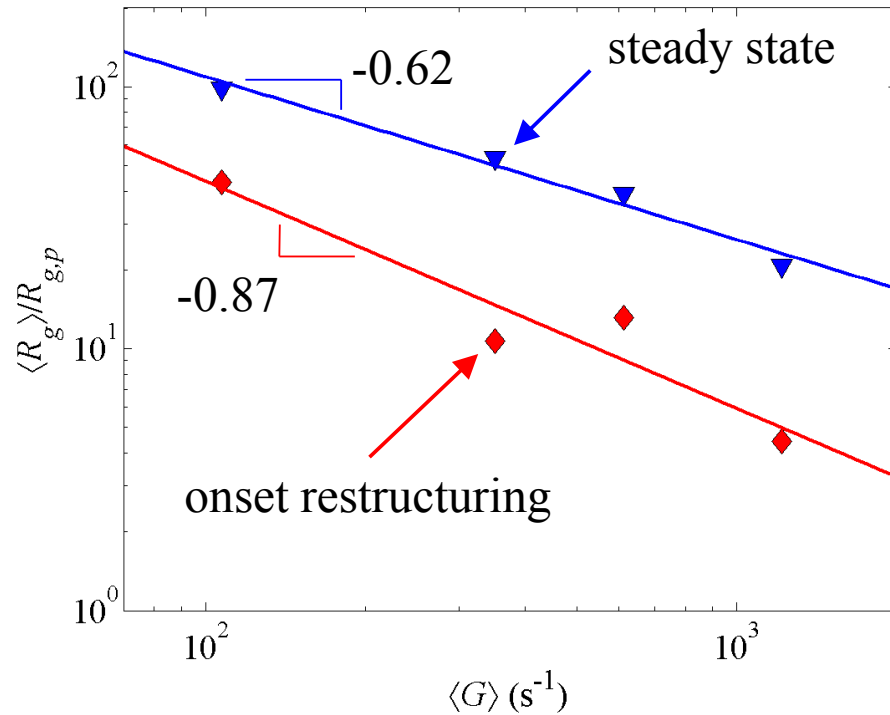
# Onset and duration of restructuring



- The onset of restructuring is more sensitive to the shear rate  $G$

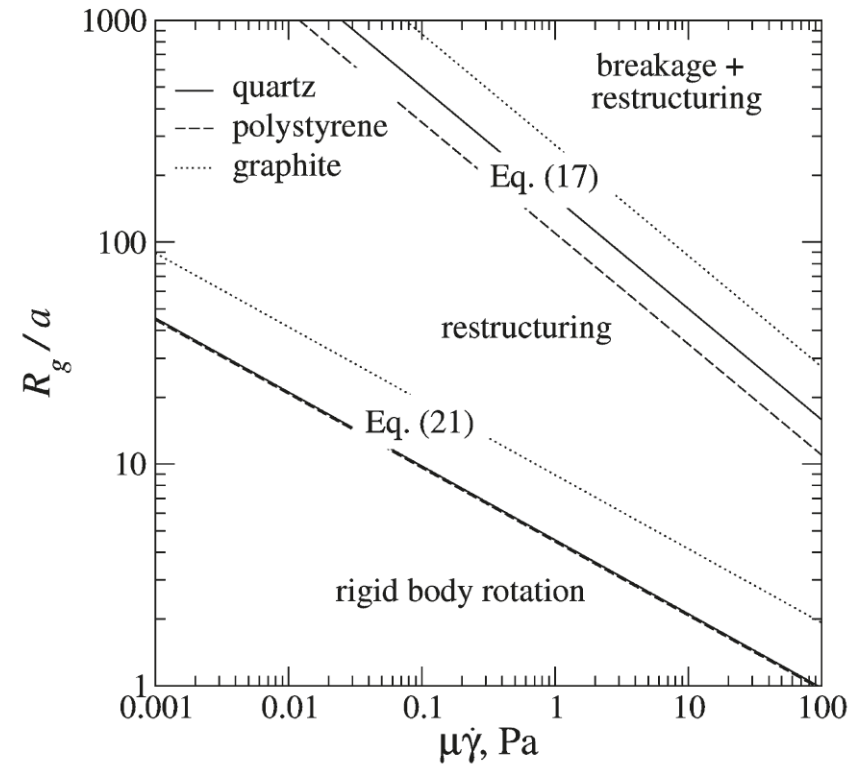
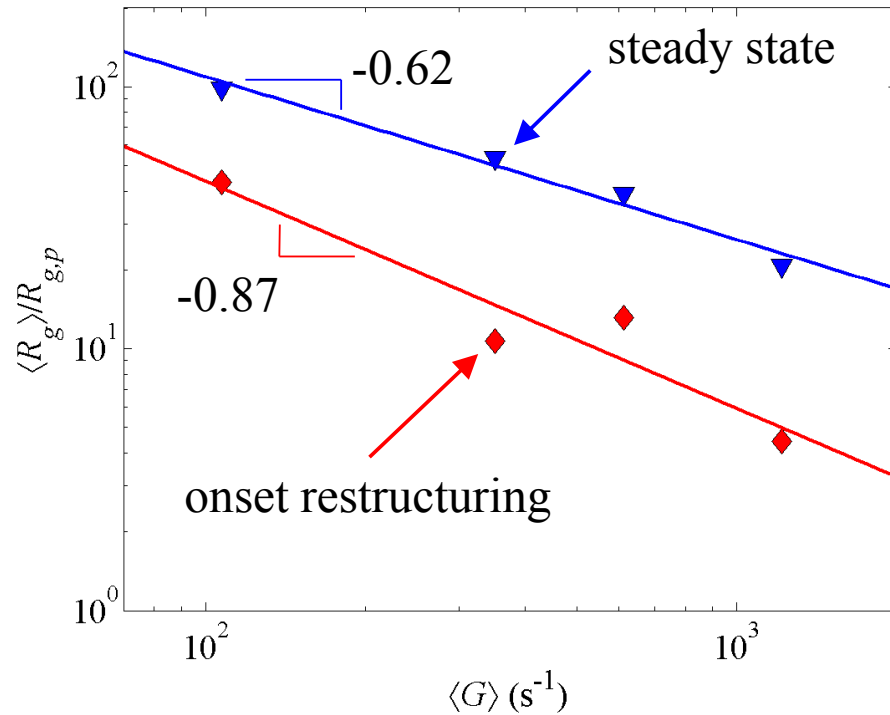


# Onset and duration of restructuring



- The onset of restructuring is more sensitive to the shear rate  $G$
- The duration of restructuring scales with the shear rate  $G$

# Onset and duration of restructuring



**Figure 12.** Regime map for restructuring/breakage for fully destabilized suspensions of different materials;  $a = 0.5 \mu\text{m}$ . The physical data for the considered materials ( $E, \nu, \sigma$ ) were taken from Dominik and Tielens.<sup>21</sup>



ROYAL INSTITUTE  
OF TECHNOLOGY

# Outline

- Introduction
- Structure of aggregates: aggregate fractal dimension
- Strategy for exploring restructuring
- Population balance model
- Breakup model
- Results
- **Conclusions**



ROYAL INSTITUTE  
OF TECHNOLOGY

# Conclusions

- Restructuring, in terms of the evolution of the fractal dimension, has been explored by fitting a PBE model to a set of experimental data.
- Restructuring sets in as the aggregates reach a certain size, and it is finished before they reach the steady state size.
- The aggregate size for the onset of restructuring depends stronger than predicted by considering only stress induced restructuring. This hints to collision induced restructuring.
- Restructuring is relatively fast and its duration scales approximately with the shear rate.
- Macromixing and variations of  $d_f$  among the aggregates might become relevant at high stirring speeds.



ROYAL INSTITUTE  
OF TECHNOLOGY

# Acknowledgments

- Miroslav Soos, ETH Zurich
- COST Action MP0805 "Particles in Turbulence"
- Swedish Research Council (VR)